The Solar Energy Book

written by

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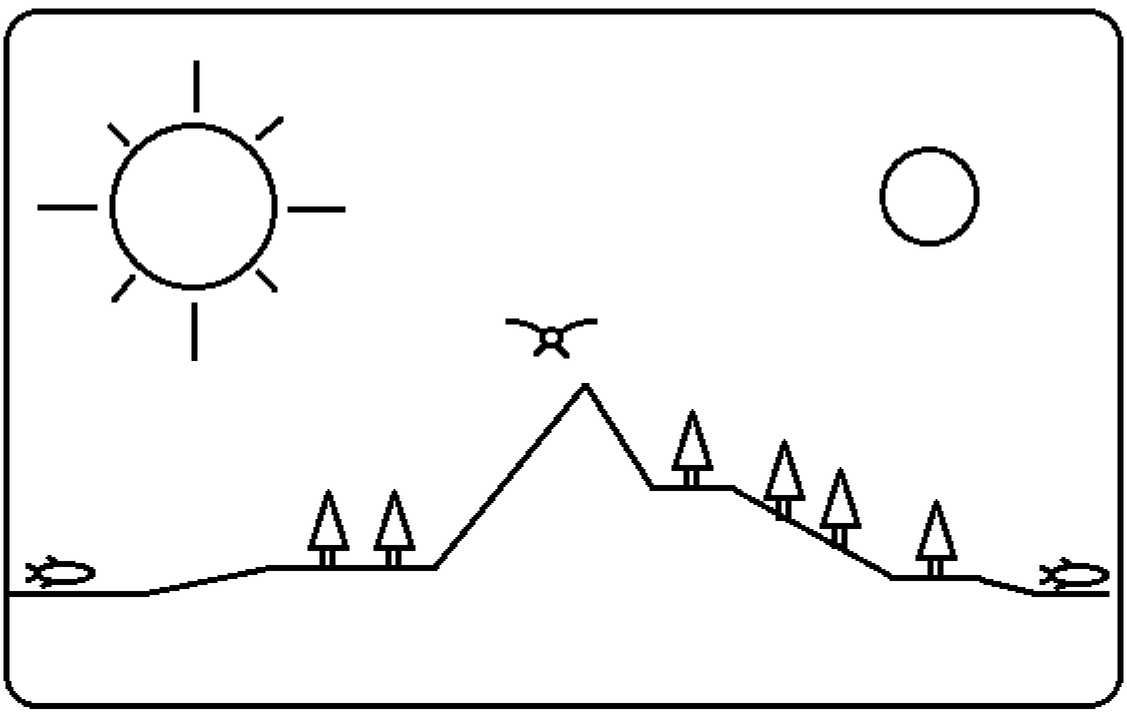


Table of Contents

Introduction

The Earth is Our Mother,

The Sun is our Father,

All of Our Material Possessions

are Derived from the Earth and the Sun.

Such was the Way of the Forgotten Eden.

Such will be the Way when we Return.

This is an attempt to write a definitive book on the subject of solar energy. This is not a book about technology. Technology is by nature unstable. This is a book about calculating for solar energy applications as found in the default natural world. This is a book about natural philosophy.

This book begins with a look at spherical trigonometry. Much of the mathematics is derived from spherical trigonometry. Everyone who reads this book will have their own purposes. Because of this, many seemingly extraneous formulae have been provided.

Chapter II will briefly go over the elementary orbital mechanics with respect to the annual variations in the solar constant. This subject is an important background for understanding the nature of solar energy.

Chapter III will delve into the Terrestrial and Celestial Spheres in order to provide a background for the calculations.

Chapters IV through VIII will introduce the formulae, processes, and equations for calculating the positions, times, powers, and energies of the Sun given a Terrestrial latitude for the site and a Celestial declination for the Sun.

Chapter IX will take a look at the seasons of the Earth with respect to the obliquity of the polar axis of the Earth. It will cover the calculations for the declination of the Sun and the effected areas of the Earth. Some observations will be included.

Chapters X through chapter XII take a brief look at three of the inorganic derivatives of solar energy. If the energy of the Sun is represented by fire; then the ordering of these three derivatives will be represented by Earth, Wind, and Water respectfully. In the ancient order of the elements;“fire” came first, and out of the “fire” came “Earth”, “Wind”, and “Water.”

Chapter XIII is a brief summary of the link between the energy of the Sun and organic life on the Earth.

Chapter XIV is brief look at the primordial building blocks of organic life on the Earth.

The final concluding chapter concerns itself with the process of doing a proper environmental survey of the site. This is a needful obligation.

Chapter I

A Foundation in

Spherical Trigonometry

As Pertaining to Solar Energy and Navigational Calculations

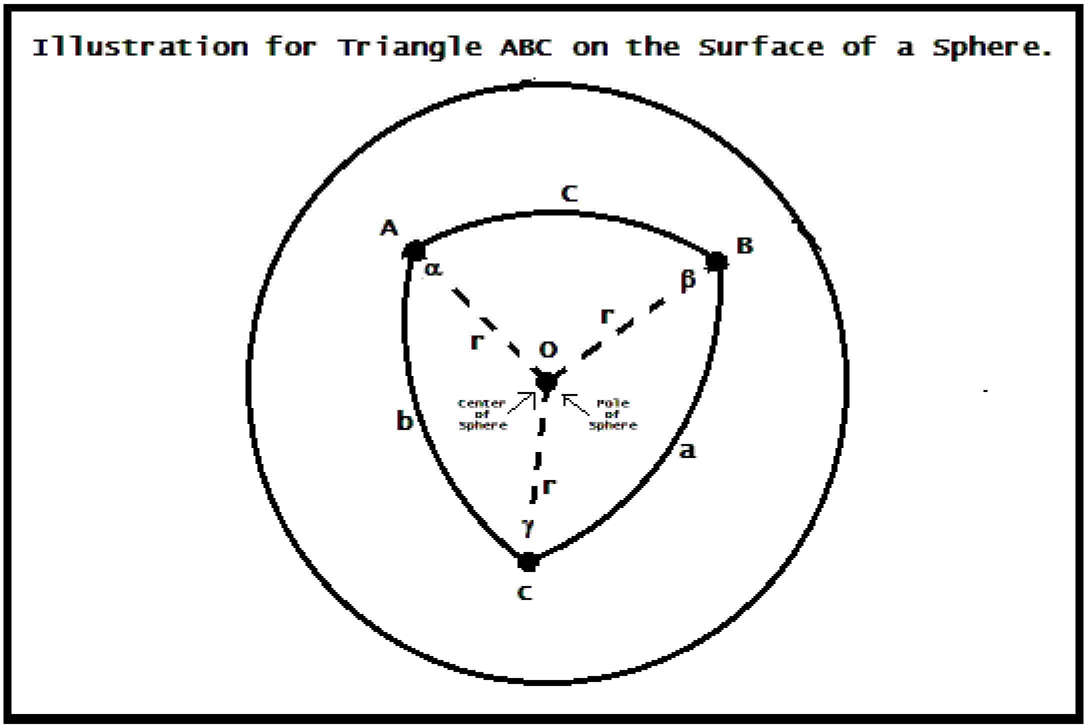


Illustration for Triangle ABC on the Surface of a Sphere.

In solar Energy calculations it is necessary to know the apparent position of the Sun with respect to a level unobstructed plane at the locale of the observer. It is equally necessary to know the position of the Sun with respect to an arbitrary slope of interest at the locale of the observer. These two cases both involve astronomical and navigational calculations. All of these calculations are done by using Spherical Trigonometry as pertaining to the curved surface of a sphere.

Illustration for Triangle ABC on the Surface of a Sphere.

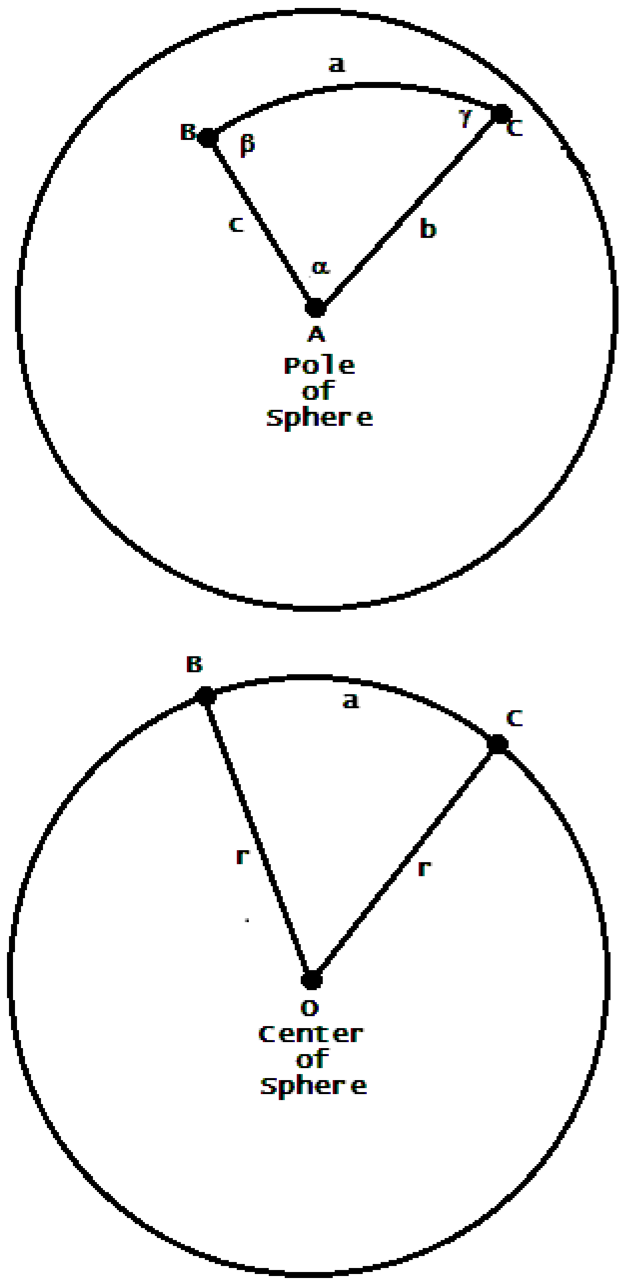


Illustration for Triangle ABC on the Surface of a Sphere.

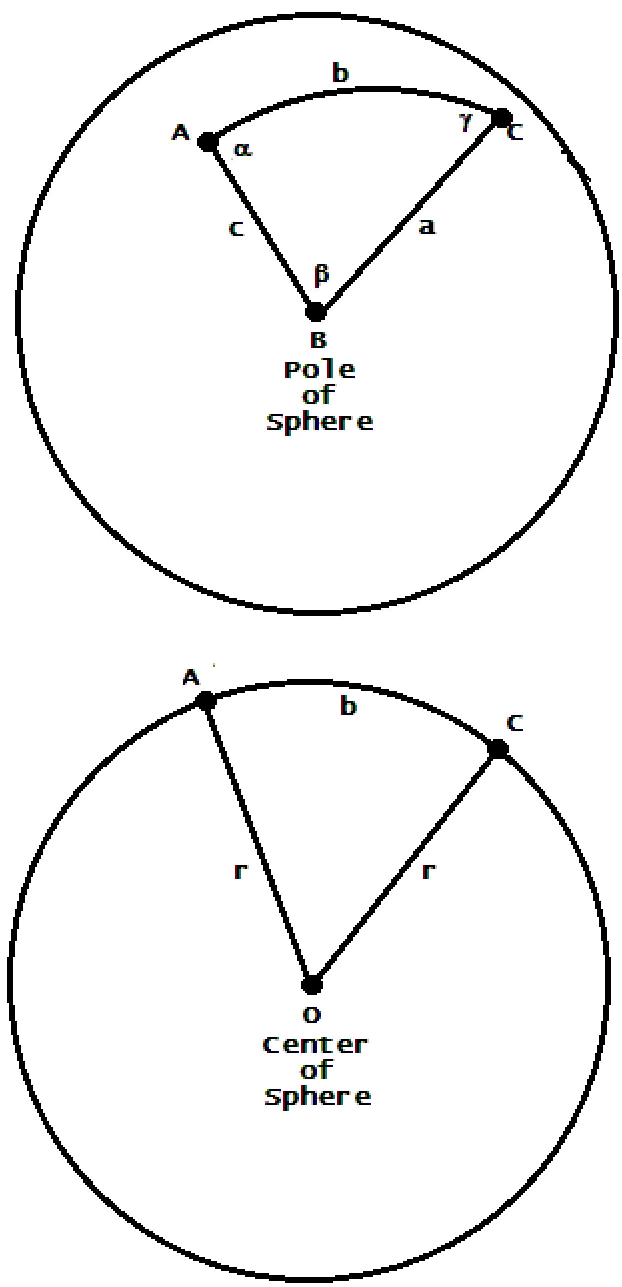
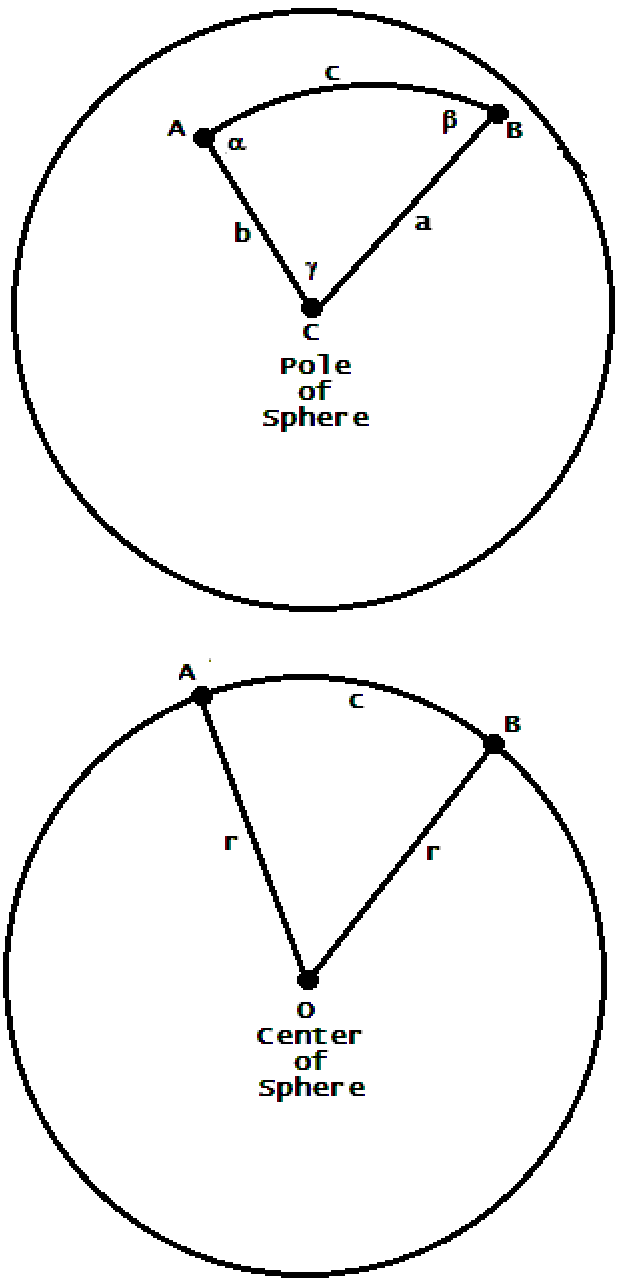


Illustration for Triangle ABC on the Surface of a Sphere.



Examine the preceding four plates. The following discussion will be in reference to them.

Spherical trigonometry as pertaining to the curved surface of a sphere is used to calculate the angular values for a “great circle” triangle enclosed within three points. The calculations are for both the arc-length of the three “sides” as well as the angles between any two adjacent sides. There is a fourth point at the center of the sphere. All four points for a tetrahedron of sorts. In the illustrations these “points” are indicated by uppercase English letters i.e. (A,B,C.O).

Any two points on the curved surface of a sphere will form a sector of a circle with respect to the center of the sphere. Given a unit radius of one with respect to the center of the sphere, the arc-length of the surface curvature will be measured as an angle in terms of radians with respect the center of the sphere. This arc-length is regarded as a “side” in spherical trigonometry. In the illustrations a “side” is denoted by a lower case English letter i.e. (a,b,c). Side [a] is opposite point [A] and point [O]. Side [b] is opposite point [B] and point [O]. Side [c] is opposite point [C] and point [O].

A “pole” is defined as the intersection of a line drawn from the center of the sphere through any of the three points of a triangle on the surface of the sphere. Each point on the surface of the sphere is a pole unto itself. The “angles” represent the azimuth separations of any two sides with respect to a particular “pole.” In the illustrations the angles are denoted by lower case Greek letters i.e. (α,β,γ). Angle [α] is opposite side [a]. Angle [β] is opposite side [b]. Angle [γ] is opposite side [c].

All of the arc-lengths of the sides and the surface angles are measured as radians with respect to the radius - (1 radian unit = radius) - of the body.

The Terrestrial Sphere is the surface of the Earth on which we dwell. By convention we have divided its surface into imaginary lines of latitude and longitude.

The lines of latitude form equally spaced circles of varying radii. They are all parallel, never converging. At the North and South poles they have a radius of zero. From the poles they expand outward to the Equator. At the Equator they reach their maximum radius of one Earth radius. By convention the lines of latitude are measured from the Equator. By convention the Equator is assigned a latitude of [0]. By convention, the North Pole is assigned a value of [$+\pi/2$]. By convention, the South Pole is assigned a value of [$-\pi/2$]. The lines of latitude are commonly referred to as ‘parallels’.

The lines of longitude are all of a common radius equal to the radius of the Earth. The lines of longitude all converge at the North and South poles of the Earth. The lines of longitude are commonly referred to as ‘meridians’. By convention, the meridians are measured from an arbitrary ‘prime meridian’ that a ruling power has chosen. All longitudes east of the prime meridian are assigned a positive [+] value. All longitudes west of the prime meridian are assigned a negative [-] value. In solar energy applications the ‘prime meridian’ is the meridian of the observer.

‘North’ represents the cardinal direction where the ‘master’ race has its origin. ‘South’ represents the cardinal direction where the ‘subservient’ races dwell. ‘East’ always represents the cardinal direction where the Sun rises. ‘West’ always represents the cardinal direction where the Sun sets. These are by either spoken or unspoken agreements.

The Celestial Sphere is sometimes referred to as the ‘Sphere of Heaven’ or the ‘Dome of Heaven’. It is the unreachable realm of the Stars, Planets, Comets, Gods, and Angels. The Celestial Sphere may take on either of two forms.

One form of the Celestial Sphere is taken from the center of the Earth. The Earth revolves on its polar axis once every sidereal day. This is the definition of the sidereal day. The Terrestrial North and South Poles are projected from the center of the Earth onto the Celestial Sphere as the North and South Celestial Poles respectively. As the Earth revolves from West to East, the Celestial Sphere appears to revolve from East to West, carrying with it all of the Celestial bodies including the Sun. The grid system for the Terrestrial Sphere is projected onto the Celestial Sphere. The latitude of the Terrestrial Sphere is projected onto the Celestial Sphere as the declination. The longitude of the Terrestrial Sphere is projected onto the Celestial Sphere as the right ascension.

The other form of the Celestial Sphere is taken from an observer on the surface of the Earth. This is a perpendicular projection which may or may not pass through the center of the Earth. This form is widely used in navigational applications and for an arbitrary slope in solar energy applications.

The Sun appears to move in the Celestial Sphere. Over a period of one year it will complete a circle of the Celestial meridians. Over the same year, the Sun will rise, fall, and rise again in the Celestial Sphere.

This has been just and introduction to the intricacies of astronomical calculations and navigational calculations as a foundation. Now let us continue on with the Spherical Trigonometric equations as pertaining to three points on the surface of a sphere. The symbols will all be in reference to the preceding illustrations.

Source: $\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$

To isolate cos(b) a quadratic equation is required due to having both a cos(b) and a sin(b) in the source equation. This process is shown in [1 - 12]. [11] and [12] are the final solution.

1: $\cos(a) - \cos(b)\cdot\cos(c) = \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$

2: $(\cos(a) - \cos(b)\cdot\cos(c))^2 = (\sin(b)\cdot\sin(c)\cdot\cos(\alpha))^2$

3: $\cos(a)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(b)^2\cdot\cos(c)^2$

$= \sin(b)^2\cdot\sin(c)^2\cdot\cos(\alpha)^2$

4: $\sin(b)^2 = 1 - \cos(b)^2$ 5: $\sin(c)^2 = 1 - \cos(c)^2$

6: $\cos(a)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(b)^2\cdot\cos(c)^2$

$= \left(1 - \cos(b)^2\right)\cdot\left(1 - \cos(c)^2\right)\cdot\cos(\alpha)^2$

7: $\cos(a)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(b)^2\cdot\cos(c)^2$

$= \left(1 - \cos(b)^2 - \cos(c)^2 + \cos(b)^2\cdot\cos(c)^2\right)\cdot\cos(\alpha)^2$

8: $\cos(a)^2 - 2\cdot\cos(a)\cdot\cos(b)\cdot\cos(c) + \cos(b)^2\cdot\cos(c)^2$

$= \cos(\alpha)^2 - \cos(b)^2\cdot\cos(\alpha)^2 - \cos(c)^2\cdot\cos(\alpha)^2 + \cos(b)^2\cdot\cos(c)^2\cdot\cos(\alpha)^2$

9: $\cos(b)^2\cdot\cos(c)^2\cdot\cos(\alpha)^2 - \cos(b)^2\cdot\cos(\alpha)^2 - \cos(b)^2\cdot\cos(c)^2$

$+ \cos(b)\cdot 2\cdot\cos(a)\cdot\cos(c)$

$+ \cos(\alpha)^2 - \cos(c)^2\cdot\cos(\alpha)^2 - \cos(a)^2$

10: $\cos(b)^2\cdot\left(\cos(c)^2\cdot\cos(\alpha)^2 - \cos(\alpha)^2 - \cos(c)^2\right)$

$+ \cos(b)\cdot(2\cdot\cos(a)\cdot\cos(c))$

$+ \cos(\alpha)^2 - \cos(c)^2\cdot\cos(\alpha)^2 - \cos(a)^2$

11: $A = \cos(c)^2 \cdot \cos(\alpha)^2 - \cos(\alpha)^2 - \cos(c)^2$

$B = 2 \cdot \cos(a) \cdot \cos(c)$

$C = \cos(\alpha)^2 - \cos(c)^2 \cdot \cos(\alpha)^2 - \cos(a)^2$

12: $\cos(b) = \dfrac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$ $\cos(b) = \dfrac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$

Finding the equation for cos(c) is identical to the solution for finding the equation fpr cos(b). There is no need to repeat the quadratic process. All that is needed is to swap cos(b) for cos(c) and cos(c) for cos(b). This is shown in [13] through [15] in the final solution.

13: $\cos(c)^2 \cdot \left(\cos(b)^2 \cdot \cos(\alpha)^2 - \cos(\alpha)^2 - \cos(b)^2\right)$

$+ \cos(c) \cdot (2 \cdot \cos(a) \cdot \cos(b))$

$+ \cos(\alpha)^2 - \cos(c)^2 \cdot \cos(\alpha)^2 - \cos(a)^2$

14: $A = \cos(b)^2 \cdot \cos(\alpha)^2 - \cos(\alpha)^2 - \cos(b)^2$

$B = 2 \cdot \cos(a) \cdot \cos(b)$

$C = \cos(\alpha)^2 - \cos(b)^2 \cdot \cos(\alpha)^2 - \cos(a)^2$

15: $\cos(b) = \dfrac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$ $\cos(b) = \dfrac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$

Finding the equation for cos(α) is more straight-forward process.

16: $\cos(a) - \cos(b) \cdot \cos(c) = \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$

17: $\cos(\alpha) = \dfrac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}$

The preceding “source equation” is one of the two law of cosines formulas that are commonly supplied by the mathematical handbooks. The proofs for the internal variants have been given. The proofs for the “source” equation have not. The “source equation” and its variants have their patterns. Here is the pattern for the “source equation.”

$$\cos(a) = \cos(b) * \cos(c) + \sin(b) * \sin(c) * \cos(\alpha)$$

This is a side-side-side-angle form. Two sides and one angle are known. The remaining side is unknown. The remaining unknown side is opposite the known angle. Here are the explanations of the six elements from left to right.

Cos(a): Cos(a) represents the cosine of the unknown arc-angle subtended from the center of the sphere.

Cos(b): Cos(b) represents the cosine of one of the two known arc-angles subtended from the center of the sphere.

Cos(c): Cos(c) represents the cosine of the second of the two known arc-angles subtended from the center of the sphere.

Sin(b): Sin(b) represents the sine of one of the two known arc-angles subtended from the center of the sphere.

Sin(c): Sin(c) represents the sine of the second of the two known arc-angles subtended from the center of the sphere.

Cos(α): Cos(α) represents the cosine of the surface angle opposite side [a] subtended between side [b] and side [c].

Using this pattern; two more “source equations“ may be derived as well as their three variants. This will result in 12 fundamental equations.

$$\cos(b) = \cos(a) * \cos(c) + \sin(a) * \sin(c) * \cos(\beta)$$

$$\cos(c) = \cos(a) * \cos(b) + \sin(a) * \sin(b) * \cos(\gamma)$$

The following two plates are a summary of the first 12 formulas concerning side-side-side-angle formulas and their internal variants. The original source formula from the handbook had as follows: Side [a], Side [b], Side [c], Angle [α] opposite Side [a]. These formulae can be substituted into parts of other formulae to obtain more choices for the situation at hand.

1a: $\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$

1b: $A = \cos(c)^2\cdot\cos(\alpha)^2 - \cos(\alpha)^2 - \cos(c)^2$

$B = 2\cdot\cos(a)\cdot\cos(c)$

$C = \cos(\alpha)^2 - \cos(c)^2\cdot\cos(\alpha)^2 - \cos(a)^2$

$\cos(b) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad \cos(b) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

1c: $A = \cos(b)^2\cdot\cos(\alpha)^2 - \cos(\alpha)^2 - \cos(b)^2$

$B = 2\cdot\cos(a)\cdot\cos(b)$

$C = \cos(\alpha)^2 - \cos(b)^2\cdot\cos(\alpha)^2 - \cos(a)^2$

$\cos(c) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad \cos(c) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

1d: $\cos(\alpha) = \dfrac{\cos(a) - \cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)}$

2a: $\cos(b) = \cos(a)\cdot\cos(c) + \sin(a)\cdot\sin(c)\cdot\cos(\beta)$

2b: $A = \cos(c)^2\cdot\cos(\beta)^2 - \cos(\beta)^2 - \cos(c)^2$

$B = 2\cdot\cos(b)\cdot\cos(c)$

$C = \cos(\beta)^2 - \cos(c)^2\cdot\cos(\beta)^2 - \cos(b)^2$

$\cos(a) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad$ $\cos(a) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

2c: $A = \cos(a)^2\cdot\cos(\beta)^2 - \cos(\beta)^2 - \cos(a)^2$

$B = 2\cdot\cos(a)\cdot\cos(b)$

$C = \cos(\beta)^2 - \cos(a)^2\cdot\cos(\beta)^2 - \cos(b)^2$

$\cos(c) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad$ $\cos(c) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

2d: $\cos(\beta) = \dfrac{\cos(b) - \cos(a)\cdot\cos(c)}{\sin(a)\cdot\sin(c)}$

3a: $\cos(c) = \cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma)$

3b: $A = \cos(b)^2\cdot\cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

$B = 2\cdot\cos(b)\cdot\cos(c)$

$C = \cos(\gamma)^2 - \cos(b)^2\cdot\cos(\gamma)^2 - \cos(c)^2$

$\cos(a) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad$ $\cos(a) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

3c: $A = \cos(a)^2\cdot\cos(\gamma)^2 - \cos(\gamma)^2 - \cos(a)^2$

$B = 2\cdot\cos(a)\cdot\cos(c)$

$C = \cos(\gamma)^2 - \cos(a)^2\cdot\cos(\gamma)^2 - \cos(c)^2$

$\cos(b) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad$ $\cos(b) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

3d: $\cos(\gamma) = \dfrac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)}$

Here are the three cases for an angle-angle-angle-side situation where an unknown angle (on the left) lies opposite a known side (on the right). The first of these three was derived from a mathematical handbook. The other two were adapted to the pattern of the first. Originally the two terms to the right were in the reverse order, but it is not considered good mathematical practice to lead an expression with a negative (-) sign whenever there is a reasonable option.

cos(α) = sin(β) * sin(γ) * cos(a) - cos(β) * cos(γ)

cos(β) = sin(α) * sin(γ) * cos(b) - cos(α) * cos(γ)

cos(γ) = sin(α) * sin(β) * cos(c) - cos(α) * cos(β)

The following two plates display the 12 internal variants of these three equations.

4a: $\cos(\alpha) = \sin(\beta)\cdot\sin(\gamma)\cdot\cos(a) - \cos(\beta)\cdot\cos(\gamma)$

4b: $A = \cos(\gamma)^2\cdot\cos(a)^2 - \cos(a)^2 - \cos(\gamma)^2$

$B = -(2\cdot\cos(\alpha)\cdot\cos(\gamma))$

$C = \cos(a)^2 - \cos(\gamma)^2\cdot\cos(a)^2 - \cos(\alpha)^2$

$\cos(\beta) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad \cos(\beta) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

4c: $A = \cos(\beta)^2\cdot\cos(a)^2 - \cos(a)^2 - \cos(\beta)^2$

$B = -(2\cdot\cos(\alpha)\cdot\cos(\beta))$

$C = \cos(a)^2 - \cos(\beta)^2\cdot\cos(a)^2 - \cos(\alpha)^2$

$\cos(\gamma) = \dfrac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$ $\qquad \cos(\gamma) = \dfrac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$

4d: $\cos(a) = \dfrac{\cos(\alpha) + \cos(\beta)\cdot\cos(\gamma)}{\sin(\beta)\cdot\sin(\gamma)}$

$$5a:\ \cos(\beta) = \sin(\alpha)\cdot\sin(\gamma)\cdot\cos(b) - \cos(\alpha)\cdot\cos(\gamma)$$

$$5b:\ A = \cos(\gamma)^2\cdot\cos(b)^2 - \cos(b)^2 - \cos(\gamma)^2$$

$$B = -(2\cdot\cos(\beta)\cdot\cos(\gamma))$$

$$C = \cos(b)^2 - \cos(\gamma)^2\cdot\cos(b)^2 - \cos(\beta)^2$$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A} \qquad \cos(a) = \frac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$$

$$5c:\ A = \cos(\alpha)^2\cdot\cos(b)^2 - \cos(b)^2 - \cos(\alpha)^2$$

$$B = -(2\cdot\cos(\alpha)\cdot\cos(\beta))$$

$$C = \cos(b)^2 - \cos(\alpha)^2\cdot\cos(b)^2 - \cos(\beta)^2$$

$$\cos(c) = \frac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A} \qquad \cos(c) = \frac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$$

$$5d:\ \cos(b) = \frac{\cos(\beta) + \cos(\alpha)\cdot\cos(\gamma)}{\sin(\alpha)\cdot\sin(\gamma)}$$

$$6a:\ \cos(\gamma) = \sin(\alpha)\cdot\sin(\beta)\cdot\cos(c) - \cos(\alpha)\cdot\cos(\beta)$$

$$6b:\ A = \cos(\beta)^2\cdot\cos(c)^2 - \cos(c)^2 - \cos(\beta)^2$$

$$B = -(2\cdot\cos(\beta)\cdot\cos(\gamma))$$

$$C = \cos(c)^2 - \cos(\beta)^2\cdot\cos(c)^2 - \cos(\gamma)^2$$

$$\cos(\alpha) = \frac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A} \qquad \cos(\alpha) = \frac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$$

$$6c:\ A = \cos(\alpha)^2\cdot\cos(c)^2 - \cos(c)^2 - \cos(\alpha)^2$$

$$B = -(2\cdot\cos(\alpha)\cdot\cos(\gamma))$$

$$C = \cos(c)^2 - \cos(\alpha)^2\cdot\cos(c)^2 - \cos(\gamma)^2$$

$$\cos(\beta) = \frac{-B + \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A} \qquad \cos(\beta) = \frac{-B - \sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A}$$

$$6d:\ \cos(c) = \frac{\cos(\gamma) + \cos(\alpha)\cdot\cos(\beta)}{\sin(\alpha)\cdot\sin(\beta)}$$

Now let us consider the law of sines as pertaining to a curved triangle lying on the surface of a sphere. This next plate presents the law of sines and 12 basic equations derived from it. The equations are in two sets of six.

Law of Sines: $\frac{\sin(a)}{\sin(\alpha)} = \frac{\sin(b)}{\sin(\beta)} = \frac{\sin(c)}{\sin(\gamma)}$

7a: $\sin(a) = \frac{\sin(b)\cdot\sin(\alpha)}{\sin(\beta)}$ 7b: $\sin(a) = \frac{\sin(c)\cdot\sin(\alpha)}{\sin(\gamma)}$

7c: $\sin(b) = \frac{\sin(a)\cdot\sin(\beta)}{\sin(\alpha)}$ 7d: $\sin(b) = \frac{\sin(c)\cdot\sin(\beta)}{\sin(\gamma)}$

7e: $\sin(c) = \frac{\sin(a)\cdot\sin(\gamma)}{\sin(\alpha)}$ 7f: $\sin(c) = \frac{\sin(b)\cdot\sin(\gamma)}{\sin(\beta)}$

8a: $\sin(\alpha) = \frac{\sin(\beta)\cdot\sin(a)}{\sin(b)}$ 8b: $\sin(\alpha) = \frac{\sin(\gamma)\cdot\sin(a)}{\sin(c)}$

8c: $\sin(\beta) = \frac{\sin(\alpha)\cdot\sin(b)}{\sin(a)}$ 8d: $\sin(\beta) = \frac{\sin(\gamma)\cdot\sin(b)}{\sin(c)}$

8e: $\sin(\gamma) = \frac{\sin(\alpha)\cdot\sin(c)}{\sin(a)}$ 8f: $\sin(\gamma) = \frac{\sin(\beta)\cdot\sin(c)}{\sin(b)}$

36 equations (formulae) have now been shown. The limits have not been supplied. These limits are indicated by the basic rules of mathematics.

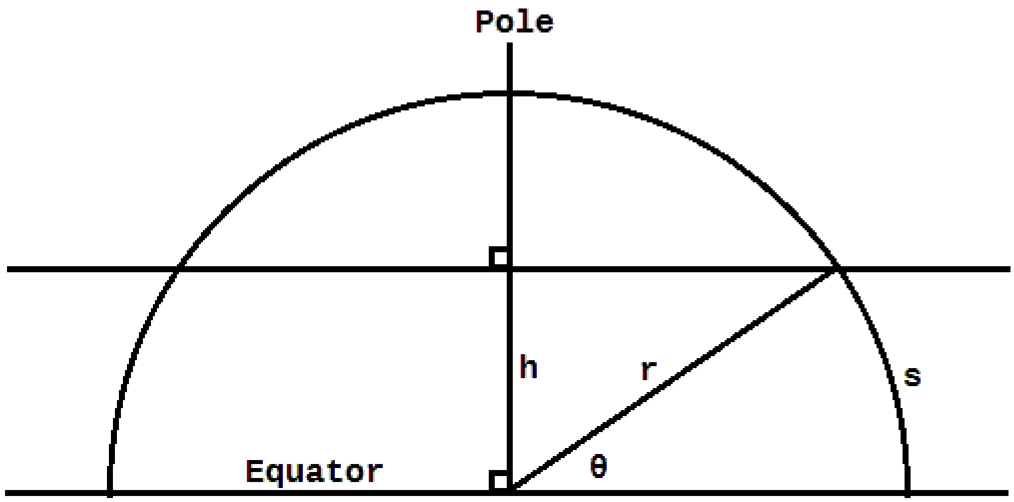
1: (0) is prohibited in the denominator.

2: A square root must be greater than or equal to (0).

3: A sine or cosine must be greater than or equal to (-1).

4: A sine or cosine must be less than or equal to (+1).

Some Useful Odds and Ends



Circumference of Circle = $2 \cdot \pi \cdot r = \frac{d}{dr}\left(\pi \cdot r^2\right)$

Area of Circle = $\pi \cdot r^2 = \int (2 \cdot \pi \cdot r)\, dr$

Surface Area of Sphere = $4 \cdot \pi \cdot r^2 = \frac{d}{dr}\left(\frac{4 \cdot \pi \cdot r^3}{3}\right)$

Volume of Sphere = $\frac{4 \cdot \pi \cdot r^3}{3} = \int \left(4 \cdot \pi \cdot r^2\right) dr$

Arc-Length [s] = $\theta \cdot r = \frac{d}{dr}\left(\frac{\theta \cdot r^2}{2}\right)$

Area of Sector of Circle = $\frac{\theta \cdot r^2}{2} = \int (\theta \cdot r)\, dr$

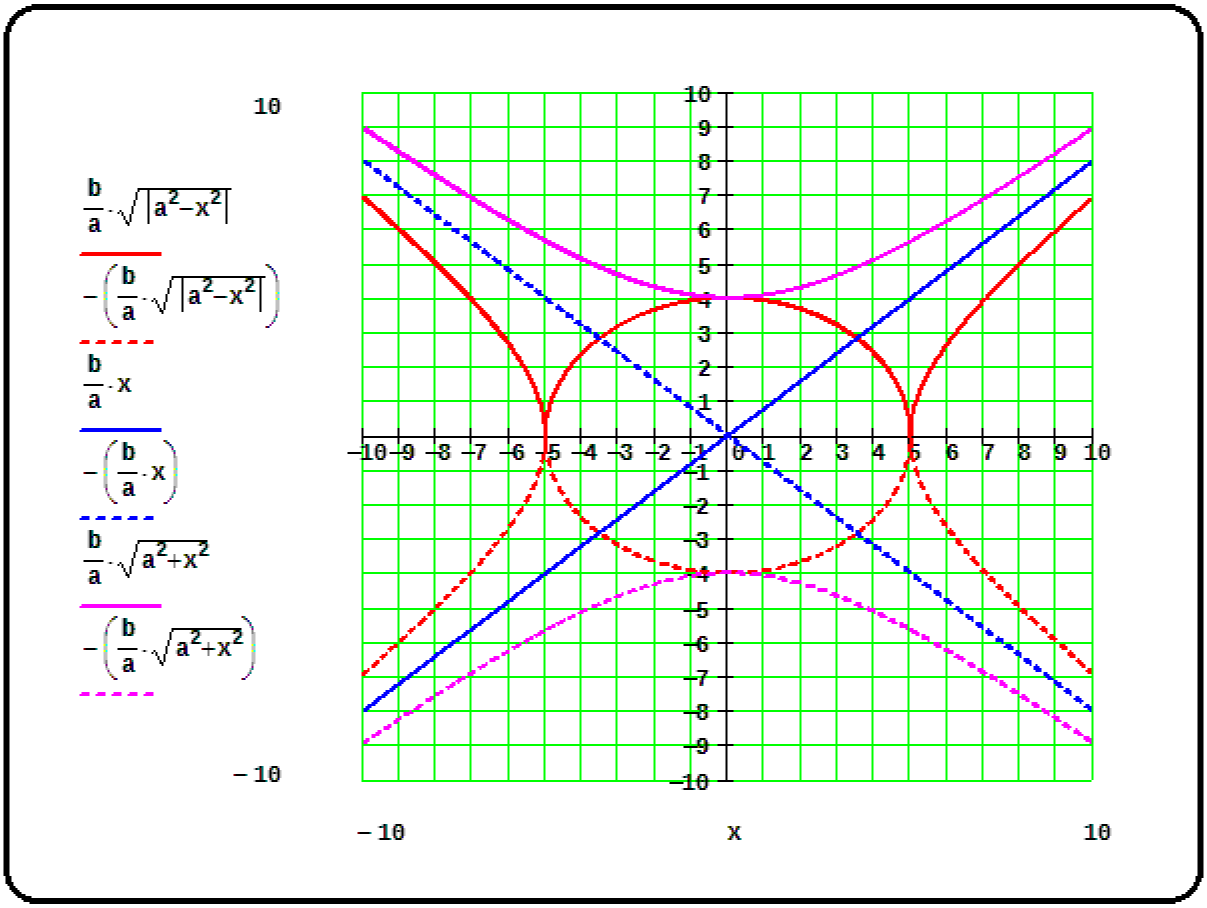
Polar Height [h] = $\sin(\theta) \cdot r$

Surface Area of Sphere within range of Height = $2 \cdot \pi \cdot h \cdot r$

Note: Angle θ is expressed in terms of radians.

Chapter II

A Foundation in Orbital Mechanics with reference to the Solar Constant



The following arguments for the orbital mechanics assumes a finite mass for the primary and an infinitesimal mass approaching zero for the orbiting body. Also assumed is a simple two-body system.

The reality in orbital mechanics is a multitude of bodies that are all revolving about one-another in a complex Celestial dance that never repeats itself. The solutions for the reality are vastly more complicated than for the simple solutions. Even if a unified formula were available to account for all the known Celestial bodies; even one of the unknown Celestial bodies would render the calculations null and void. The total mass of the unknown Celestial bodies far exceeds the total mass of the known Celestial bodies.

Thus, we must be satisfied with the simple two-body solutions for a finite primary mass and an orbiting mass of zero. The orbital database must be of as recent as possible. We must expect minor fluctuations in the reality versus the calculated expectations.

Our concern in these arguments is the solar constant. The intensity of the solar radiation varies inversely as the square of the distance from the Sun. All planets that are long term members of a solar system orbit about their primary in an elliptical orbit with the Sun located at one on the foci. All visiting Celestial bodies will approach and recede from the Sun in a hyperbolic path with the Sun at the focus. No planets have a perfect circular orbit.

We will attempt to calculate the distance between a Sun and its planets as:

(1) A function of [x] on the major axis.

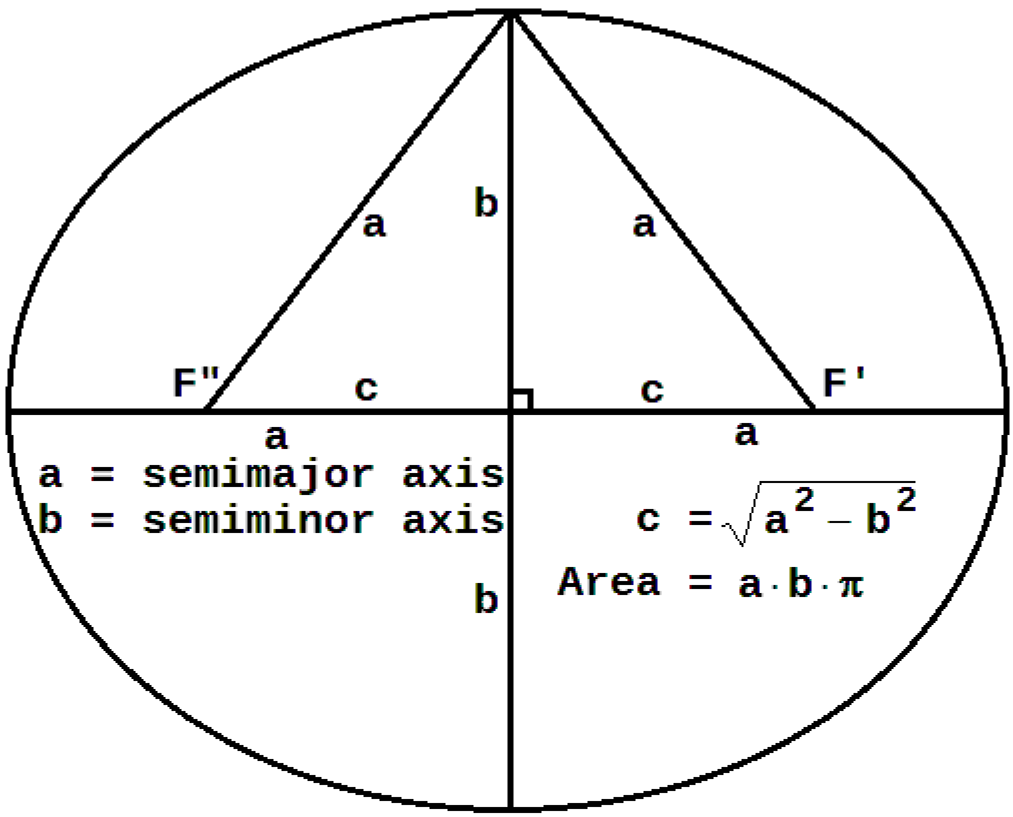
(2) A function of the angle of the planet with respect to perihelion.

(3) A function of the time of the year with respect to perihelion. This latter has complications that will be addressed.

Some Facts about the Ellipse

An ellipse is one of the “conic sections.” If a plane intersects a cone within a specified range of angles, an ellipse will form. Alternately, an ellipse may be viewed as a tilted circle, a uniformly squashed circle, or a uniformly stretched circle.

And ellipse contains two foci. A line from one focus to any given point on the perimeter and then on to the other focus will always have a combined length equal to the major axis.



The eccentricity [e] of an ellipse is the ratio of the distance from the center to the foci compared to the semi-major axis.

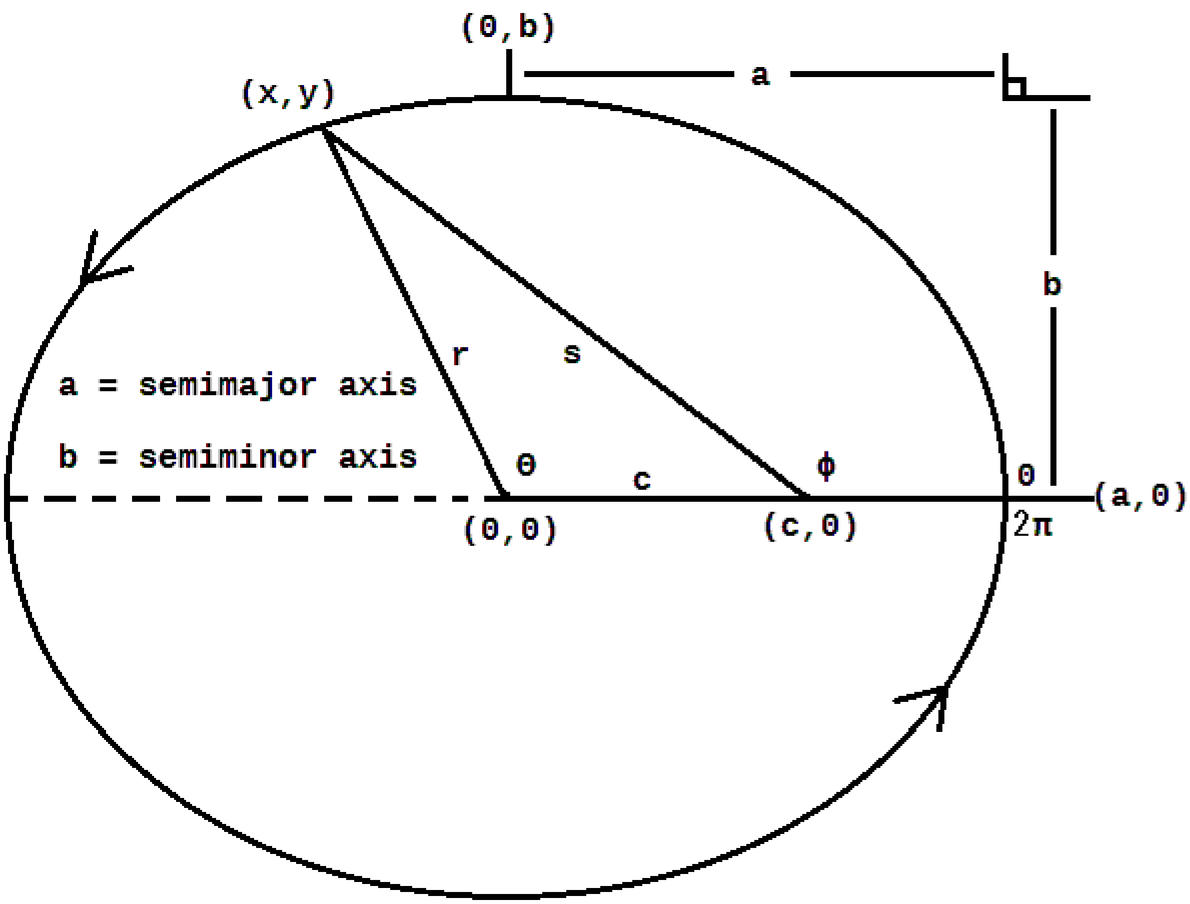
$$e = c/a$$

In an orbital ellipse, the primary occupies the place of one of the foci.

The Coordinate Equations of an Ellipse

Let us consider the coordinate algebra and trigonometry of the basic ellipse. The following illustration depicts the items of interest. The illustration is then followed by a series of 10 itemized mathematical formulas and equations. These 10 items are then shown on a sample graph. The sample graph is then followed by a series of brief explanations for each item.

It is assumed that the coordinate center of the ellipse is (0,0).



Here are 10 basic formulas and equations for the ellipse. It is assumed that the semi-major axis [a] and the semi-minor axis [b] are given.

$$1:\quad c = \sqrt{a^2 - b^2}$$

$$2:\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad b^2 \cdot x^2 + a^2 \cdot y^2 = a^2 \cdot b^2 \qquad a^2 \cdot y^2 = a^2 \cdot b^2 - b^2 \cdot x^2$$

$$3:\quad y = \pm \frac{b}{a} \cdot \sqrt{a^2 - x^2} \qquad y^2 = \frac{a^2 \cdot b^2 - b^2 \cdot x^2}{a^2} \qquad y^2 = \frac{b^2 \cdot \left(a^2 - x^2\right)}{a^2}$$

$$4:\quad \frac{d}{dx}\left(\frac{b}{a} \cdot \sqrt{a^2 - x^2}\right) = \frac{-b \cdot x}{a \cdot \sqrt{a^2 - x^2}}$$

$$5:\quad \frac{d}{dx}\left[-\left(\frac{b}{a} \cdot \sqrt{a^2 - x^2}\right)\right] = \frac{b \cdot x}{a \cdot \sqrt{a^2 - x^2}}$$

$$6:\quad \int_0^x \left(\frac{b}{a} \cdot \sqrt{a^2 - x^2}\right) dx = \frac{b}{a} \cdot \left(\frac{x \cdot \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \cdot \operatorname{asin}\left(\frac{x}{a}\right)\right)$$

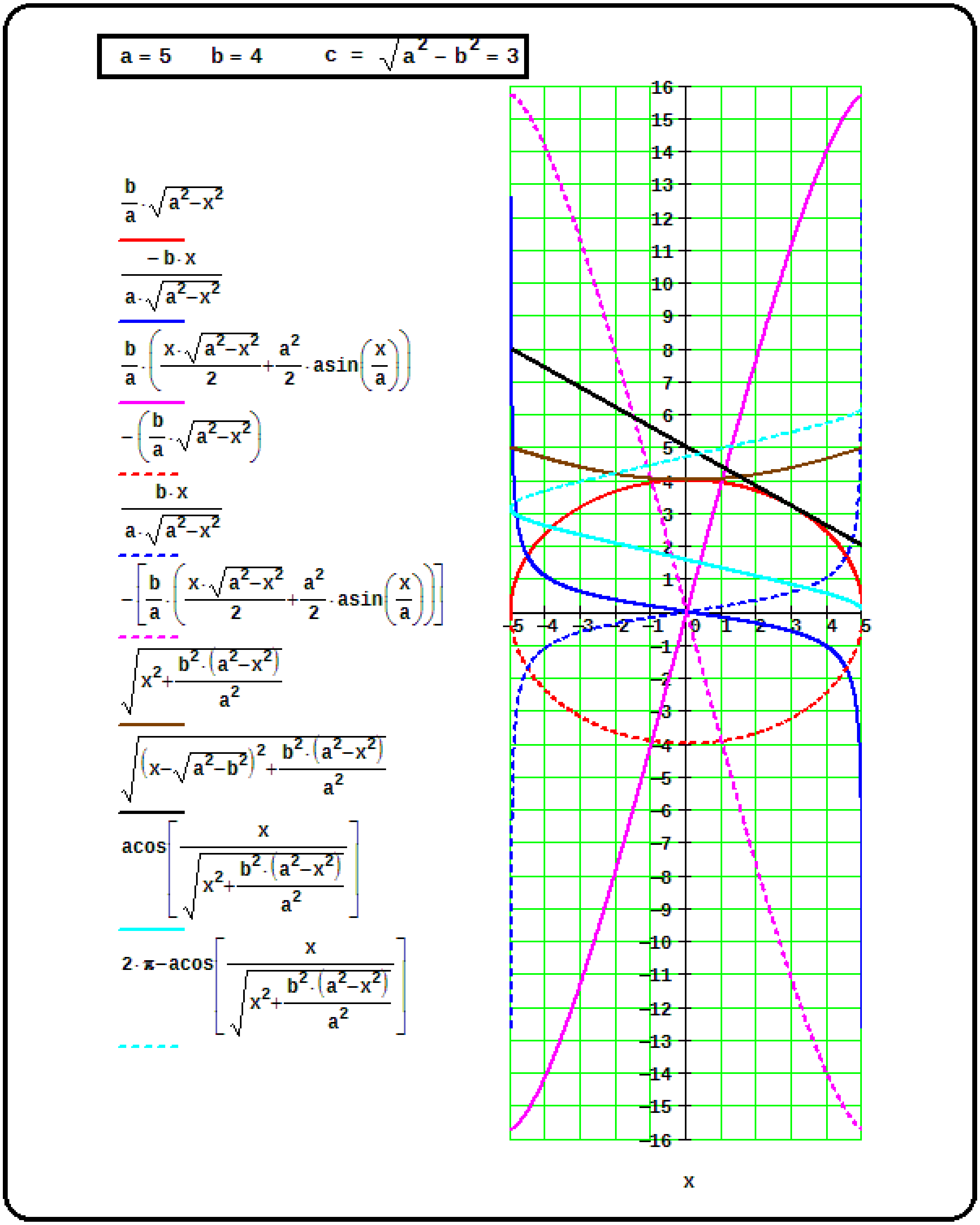
$$7:\quad r = \sqrt{x^2 + y^2} = \sqrt{x^2 + \frac{b^2 \cdot \left(a^2 - x^2\right)}{a^2}}$$

$$8:\quad s = \sqrt{(x - c)^2 + y^2} = \sqrt{\left(x - \sqrt{a^2 - b^2}\right)^2 + \frac{b^2 \cdot \left(a^2 - x^2\right)}{a^2}}$$

$$9:\quad \theta = \operatorname{acos}\left(\frac{x}{r}\right) = \operatorname{acos}\left[\frac{x}{\sqrt{x^2 + \frac{b^2 \cdot \left(a^2 - x^2\right)}{a^2}}}\right] \qquad (y \ge 0)$$

$$10:\quad \theta = 2 \cdot \pi - \operatorname{acos}\left(\frac{x}{r}\right) = 2 \cdot \pi - \operatorname{acos}\left[\frac{x}{\sqrt{x^2 + \frac{b^2 \cdot \left(a^2 - x^2\right)}{a^2}}}\right] \qquad (y < 0)$$

Here is a graph of the last 9 of the 10 items listed.



Now let us consider the explanations for these 10 items.

Given: The variable [a] represents the semi-major axis of the ellipse. The major axis of the ellipse is the name given to the long axis. The semi-major axis is half the length of the major axis.

Given: The variable [b] represents the semi-minor axis of the ellipse. The minor axis of the ellipse is the name given to the short axis. The semi-minor axis is half the length of the minor axis.

Given: The variable [x] represents the independent variable.

1: The variable [c] represents the distance between the center of the ellipse and either of the two foci.

2: This is the standard coordinate equation for the ellipse. All of the elements on the left are made to equal 1 on the right. Normally, [x] would be written as [x-h], where [h] represents the x-coordinate of the ellipse. Since [h] has been assigned a value of zero, it has been removed as extraneous for reasons of clarity. Likewise, [y] would be written as [y-k], where [k] represents the y-coordinate of the ellipse. Since [k] has been assigned a value of zero, it too has been removed as extraneous for reasons of clarity.

3: The variable [y] represents the y-coordinate of the ellipse. Here we have the y-coordinate [y] of the ellipse on the left presented as a joint function of the semi-major axis [a], the semi-minor axis [b], and the independent variable [x] on the right. There are two solutions to this equation. One is positive [+] and the other is negative [-]. The two solutions are symmetrically reflected about the semi-major axis. Both solutions are valid.

4: This is the differential equation for the positive [+] solution for +y(a,b,x). This differential represents the instantaneous slope [δy/δx] of the positive [+] perimeter for a given value of [x].

5: This is the differential equation for the negative [-] solution for -y(a,b,x). This differential represents the instantaneous slope [δy/δx] of the negative [-] perimeter for a given value of [x].

6: This is the integral equation for the positive [+] solution for +y(a,b,x). The integration is from [x=0] for a change in [x]. If two perpendicular lines to the major axis exist at [x=0] and [x=x], this integral will result in the accumulated area as a function of [x] between the two perpendicular lines, the major axis, and the positive [+] perimeter. The maximum possible return is ¼ of the total area of the ellipse. There is a negative [-] variant of this integral equation for -y(a,b,x) representing the accumulated area between the two perpendicular lines, the major axis, and the negative [-] perimeter.

7: Variable [r] represents the distance between the center of the ellipse and a specified point on the perimeter of the ellipse.

8: Variable [s] represents the distance between the left focus on the major axis and a specified point on the perimeter of the ellipse. Observe the linear progression as a function of [x].

9: Angle [θ] represents the angle subtended from the center of the ellipse between (+a,0) and (x,y) beginning at (+a,0) measured counterclockwise to (-a,0). This variable has a range [0] to [π]. It only applies to [y>0].

10: This is the alternative Angle [θ] representing the angle subtended from the center of the ellipse between (-a,0) and (x,y) beginning at (-a,0) measured counterclockwise to (+a,0). This variable has a range [π] to [2π]. It only applies to [y<0].

Note: The details of angle [ϕ] are not shown due to a lack of space in both the list of formulae and equations. Essentially it is like angle [0] except that it is subtended from the left focus. It will be detailed in the section on the orbital ellipse.

All of these equations are easily inverted and rearranged except for the integral equation.

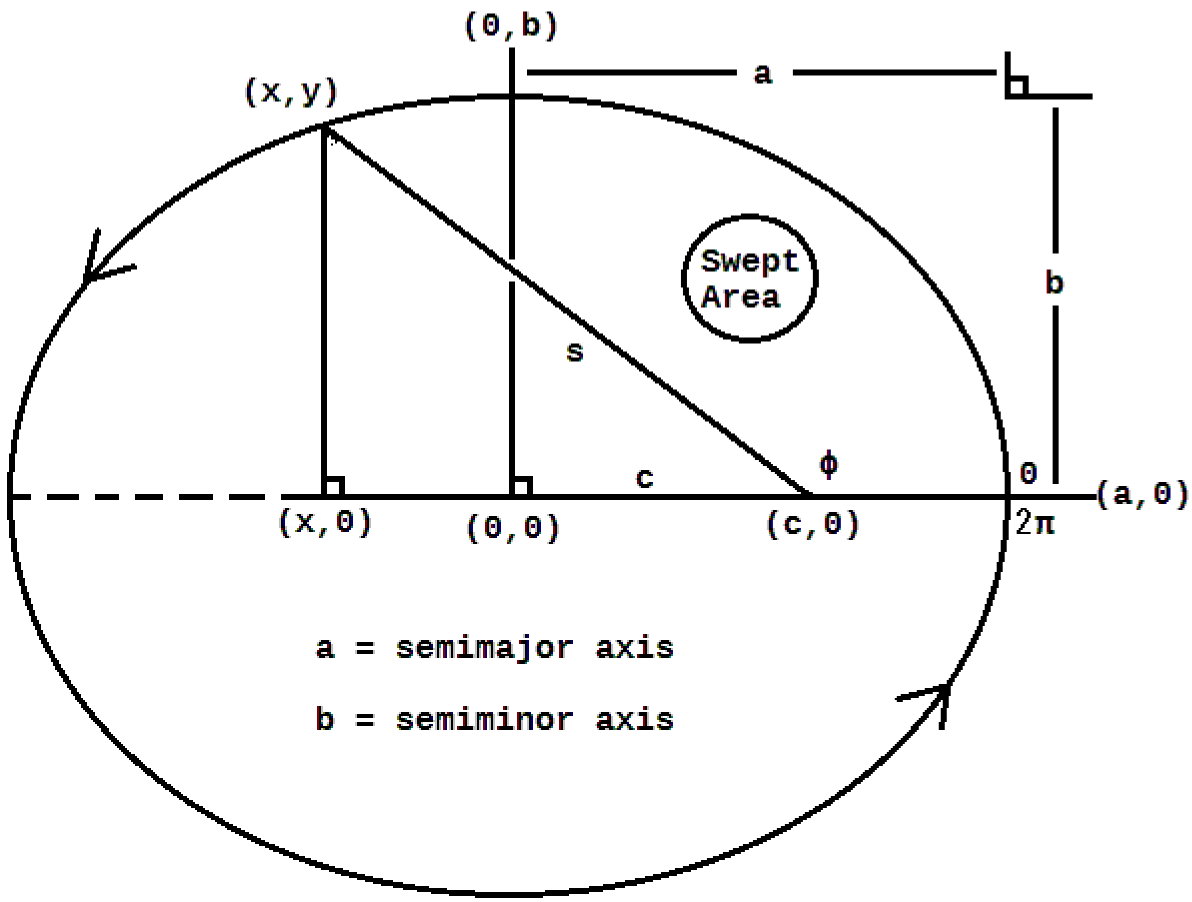
The Coordinate Equations of an Orbital Ellipse

Given two Celestial bodies revolving about one another, one of a finite mass and the other of an infinitesimal mass, the following three laws will apply to the orbit of the infinitesimal mass.

Law 1: An orbit will follow an elliptical path with the primary located at one of the two foci.

Law 2: With respect to the primary, an elliptical orbit will sweep equal areas in equal times.

Law 3: With respect to the semi-major axis, the cube of the distance will be proportional to the square of the period.



$$11:\quad s = \sqrt{(x-c)^2 + y^2} = \sqrt{(x-c)^2 + \frac{(a^2-c^2)\cdot(a^2-x^2)}{a^2}}$$

$$s^2 = (x-c)^2 + \frac{(a^2-c^2)\cdot(a^2-x^2)}{a^2} = \frac{a^2\cdot(x-c)^2 + (a^2-c^2)\cdot(a^2-x^2)}{a^2}$$

$$s^2 = \frac{a^2\cdot(x^2 - 2\cdot x\cdot c + c^2) + a^4 - a^2\cdot x^2 - a^2\cdot c^2 + c^2\cdot x^2}{a^2}$$

$$s^2 = \frac{a^2\cdot x^2 - 2\cdot a^2 x\cdot c + a^2\cdot c^2 + a^4 - a^2\cdot x^2 - a^2\cdot c^2 + c^2\cdot x^2}{a^2}$$

$$s^2 = \frac{a^4 - 2\cdot a^2\cdot x\cdot c + c^2\cdot x^2}{a^2} = \frac{(a^2 - c\cdot x)^2}{a^2}$$

$$s = \frac{a^2 - c\cdot x}{a} = \frac{a\cdot\left(a - \frac{c}{a}\cdot x\right)}{a}$$

$$12:\quad s = a - \frac{c}{a}\cdot x$$

$$13:\quad \phi = \operatorname{acos}\left(\frac{x-c}{s}\right) = \operatorname{acos}\left[\frac{x - \sqrt{a^2-b^2}}{\left(a - \frac{\sqrt{a^2-b^2}}{a}\cdot x\right)}\right] \qquad (y \ge 0,\ 0 \le \phi \le \pi)$$

$$14:\quad \phi = 2\cdot\pi - \operatorname{acos}\left(\frac{x-c}{s}\right) = 2\cdot\pi - \operatorname{acos}\left[\frac{x - \sqrt{a^2-b^2}}{\left(a - \frac{\sqrt{a^2-b^2}}{a}\cdot x\right)}\right] \qquad (y < 0,\ \pi < \phi < 2\cdot\pi)$$

$$\cos(\phi) = \frac{x-c}{s} \qquad s\cdot\cos(\phi) = x - c \qquad \left(a - \frac{c}{a}\cdot x\right)\cdot\cos(\phi) = x - c$$

$$a\cdot\cos(\phi) - \frac{c}{a}\cdot x\cdot\cos(\phi) = x - c \qquad x + \frac{c}{a}\cdot x\cdot\cos(\phi) = a\cdot\cos(\phi) + c$$

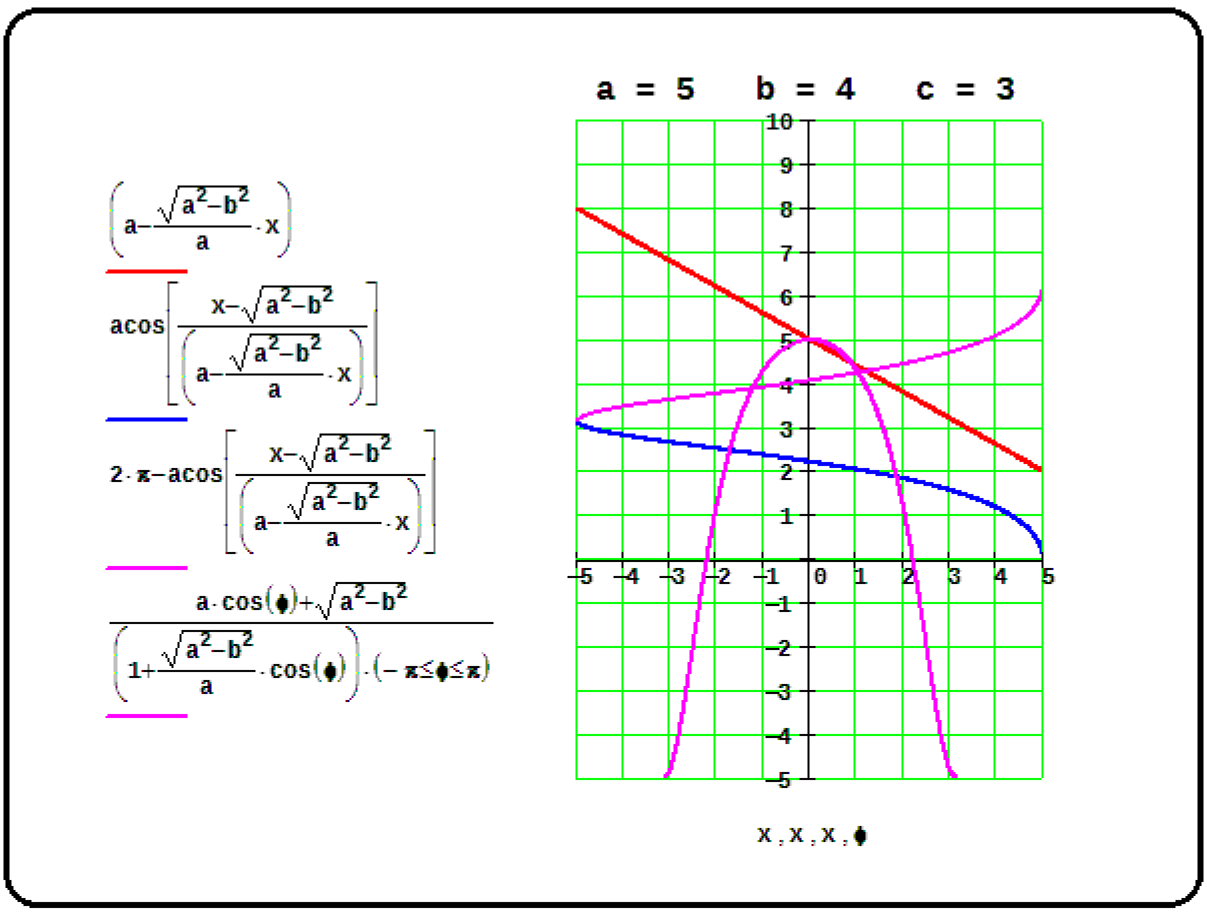
$$x\cdot\left(1 + \frac{c}{a}\cdot\cos(\phi)\right) = a\cdot\cos(\phi) + c$$

$$15:\quad x = \frac{a\cdot\cos(\phi) + c}{1 + \frac{c}{a}\cdot\cos(\phi)} = \frac{a\cdot\cos(\phi) + \sqrt{a^2-b^2}}{1 + \frac{\sqrt{a^2-b^2}}{a}\cdot\cos(\phi)}$$

The preceding formulae and equations are with respect to the illustration immediately preceding them. Below is a sample graph using these formulae and equations.

It was observed earlier that the value of [s] appeared to be a linear progression with respect to the value of [x]. [11] and [12] are the proofs of the linear progression.

Angle [ϕ] representing the angle of the orbiting body with respect to the primary, needed to be shown in its inversion as x(ϕ). This allows functions of [f(x)] to be written as functions of [f(ϕ)] by insertion.



Next we have the brief itemized explanations for [11] through [15].

In orbital mechanics it is common to use the eccentricity of the orbit rather than the semi-minor axis. The eccentricity is considerably more pronounces. The eccentricity is defined as [c/a]. Items [11] through [15] are coached in both f(a,b,x) and f(a,c,x).

11: This is the proof, (the work), for demonstrating the linear relationship between [δx] and [δs]. This linear relationship only exists when line [s] is rooted to either of the two foci. For this solution, line [s] has been rooted to the left focus at [+c,0]. It has been given as a function of s(a,c,x).

12: This is the final solution for the distance [s] representing the distance between an orbiting body and its primary as a function of [x] along the semi-major axis. It has been coached as a joint function of the semi-major axis [a], the distance from the center of the ellipse to the foci [c], and the independent x-coordinate [x].

13: Here, angle [ϕ], pronounced “phi”, represents the angle of rotation of an orbiting body about its primary with respect to its primary. The formula is given as both a joint function of ϕ(a,b,x) and ϕ(a,c,x). It is indexed to perihelion on the right and the rotation is counterclockwise. This is one of two variants of [ϕ]. This variant is only applicable when [y≥0]. It has an angular range of [0≤ϕ≤π].

14: In this case, angle [ϕ], pronounced “phi”, represents the alternate angle of rotation of an orbiting body about its primary with respect to its primary. The formula is given as both a joint function of ϕ(a,b,x) and ϕ(a,c,x). It is indexed to perihelion on the right and the rotation is counterclockwise. This is the other variant of [ϕ]. This variant is only applicable when [y<0]. It has an angular range of [π<ϕ<2π].

15: [x(ϕ)] is an essential inversion of [ϕ(x)]. In space, position [x] has no real meaning. Position [x] only has meaning in the context of the mathematical calculations. Alternately, angle [ϕ] is readily observable from either the primary or the orbiting body. It is necessary convert the angle [ϕ] to an x-coordinate position in order to carry out the mathematical computations. The return of [x] for a given independent value of [ϕ] replaces the x-coordinate in the subsequent calculations. The range of [ϕ] is unlimited [-∞<ϕ<+∞].

Now, let us continue on the calculations for the swept area of the orbit, and for the distance between the primary and the orbiting body. We will examine each of the following itemized expressions.

6: $$\int_0^x \left(\frac{b}{a}\cdot\sqrt{a^2-x^2}\right)dx = \frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cdot\mathrm{asin}\left(\frac{x}{a}\right)\right)$$

16: $$\frac{a\cdot b\cdot\pi}{4}-\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cdot\mathrm{asin}\left(\frac{x}{a}\right)\right)$$

17: $$\left(\frac{x-\sqrt{a^2-b^2}}{2}\right)\left(\frac{b}{a}\cdot\sqrt{a^2-x^2}\right)$$

18: $$\frac{a\cdot b\cdot\pi}{4}-\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cdot\mathrm{asin}\left(\frac{x}{a}\right)\right)+\left(\frac{x-\sqrt{a^2-b^2}}{2}\right)\left(\frac{b}{a}\cdot\sqrt{a^2-x^2}\right)$$

19: $$\frac{3\cdot a\cdot b\cdot\pi}{4}+\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cdot\mathrm{asin}\left(\frac{x}{a}\right)\right)-\left(\frac{x-\sqrt{a^2-b^2}}{2}\right)\left(\frac{b}{a}\cdot\sqrt{a^2-x^2}\right)$$

20: $$(\mathrm{Length\_of\_Year})\cdot\left(\frac{\mathrm{Swept\_Area\_from\_Perihelion}}{\mathrm{Area\_of\_Ellipse}}\right)$$

6: This is a re-visitation of the integral equation of the ellipse. This equation will be modified for a different perspective.

16: This expression represents the integral equation of the ellipse taken from the right at (+a,0) going towards (-a,0). The range of the return is fully half of the ellipse.

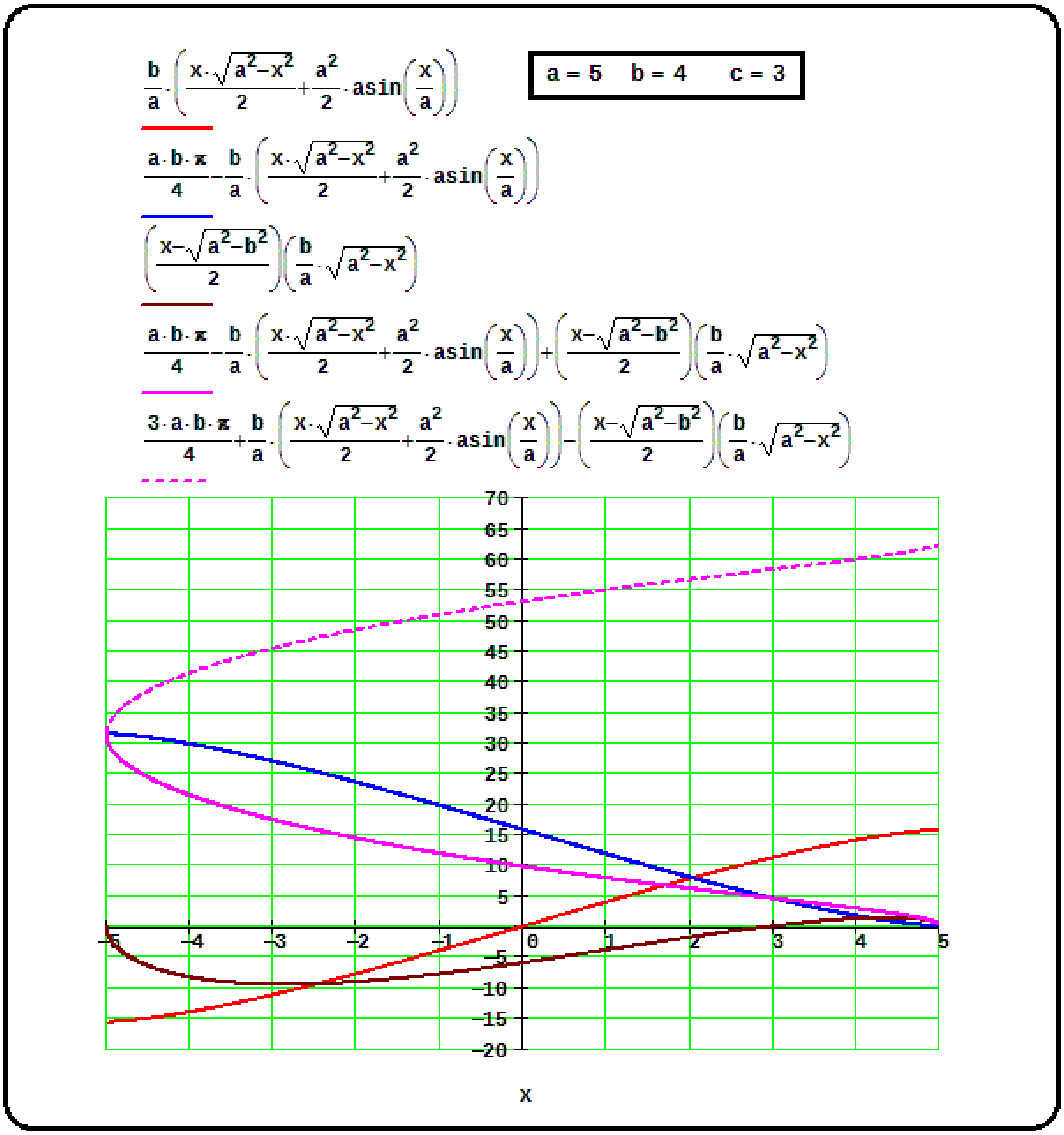
17: The integral equation of the ellipse is rooted to the center of the ellipse. Even with the adjustment from [16], it is still rooted to the center of the ellipse. This expression represents a right triangle rooted to the prime focus.

18: Here the newest right triangle is added to or subtracted from the modified integral of the ellipse as needed. It is written so that the sign shifts are automatic. It represents the swept area from right to left. It is only applicable if [y] is greater than or equal to [0] or [ϕ] is less than or equal to [π].

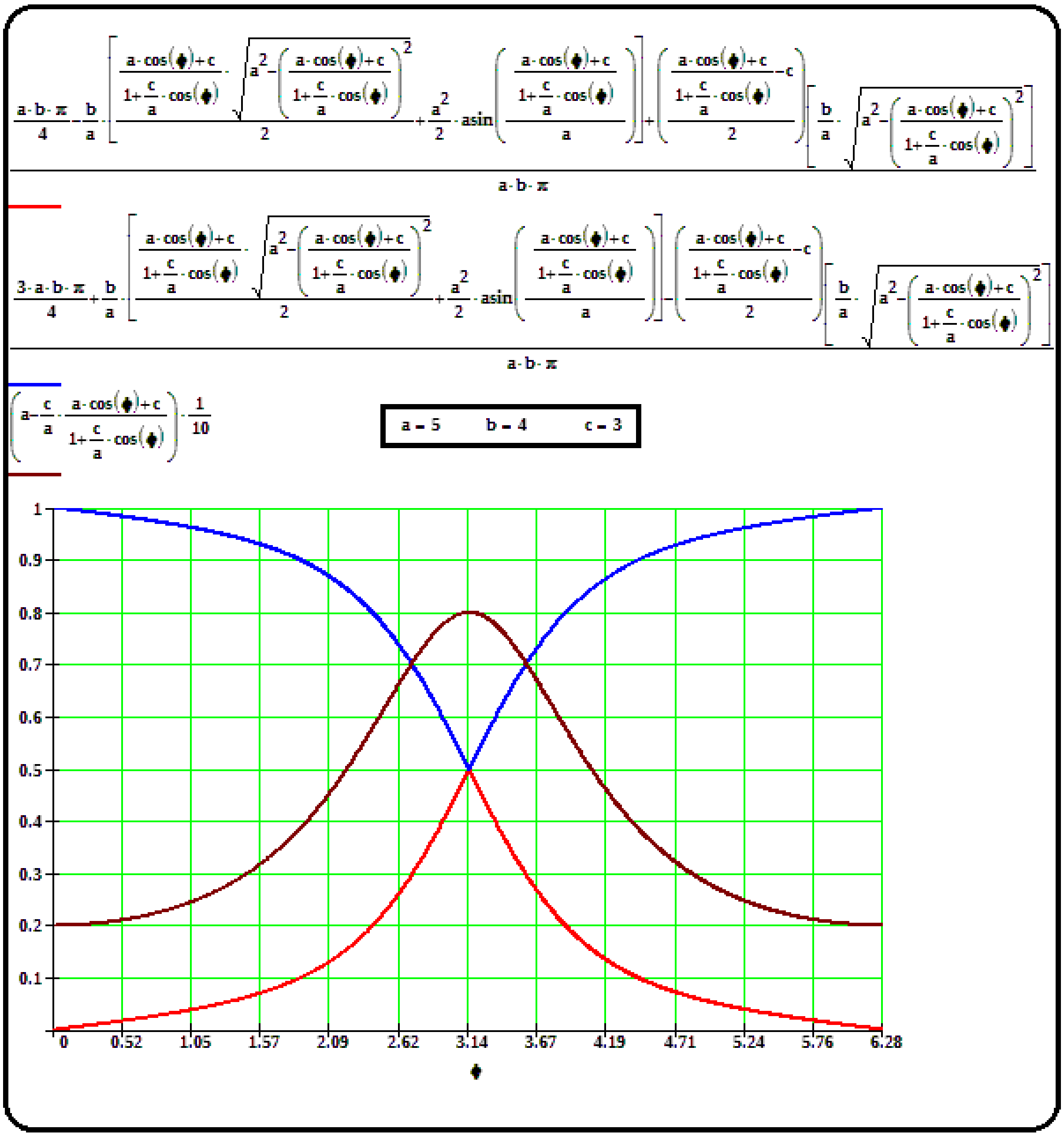
19: This expression represents the alternate solution for the swept area when [y] is less than [0] or [ϕ] is greater than [π].

20: A year is divided into divisions according to the culture that has defined the year. This is purely a matter of convenience. The portion of the whole year that has passed from perihelion is obtained by dividing the swept area by the total area of the ellipse. This results in a proportion of the year. Then the number of the divisions of the year are multiplied in.

This graph illustrates an example of [6), [16], [17], [18], and [19] in order.



This sample graph illustrates an example of the proportion of the swept area of the year as a function of the angle [ϕ]. Both of the variants are shown. Also displayed is the proportion of the length [s] from the primary and the orbiting body as a function of [ϕ] with respect to the length of the major axis.



The Solar Constant

The term “Solar Constant” is a misnomer. It refers to a baseline intensity of radiation received by a Planet from its Sun.

To begin with, a Sun is a living entity in its own right. As such it must breathe and eat. At times it will emit more radiation. At other times it will emit less radiation. Our Sun is more steady than most in this right. However, it still varies. We do not have any records of those times when our Sun showed the greatest variation in output for the simple and logical reason that the extreme deviation of solar radiation wiped out the harvest, destroyed civilization, and all that remains are myths that have been long since corrupted. This is a variation that cannot be predicted, only acknowledged and accepted.

No planet strictly revolves about its Sun. The Planets and the Sun mutually revolve about one another. However, no unified mathematical formula is possible for this variation.

Thus, we must be content with the simple elliptical formula for the distance between a Planet and its Sun. Now the intensity of the solar radiation will vary inversely as the square of the distance. Using the most recent baseline data we can calculate the simple variation in the solar radiation for the simple elliptical orbit; and accept that it will be a bit off of the reality.

$$Sol = K \cdot (a/s)^2$$

[a]= semi-major axis of orbit.

[s] = distance between Planet and Sun.

[K] = solar intensity at distance [a].

[sol] = calculated “solar constant.”

The semi-major axis of the Earth is defined as [1] astronomical unit [AU]. The eccentricity for the orbit of the Earth is [0.0167] AU. There are [365.24] solar days in a year. The solar radiation intensity at [1] AU is [1.373] kw per square meter. Here is a sample table for the orbit of the Earth including the variations in the "solar constant."

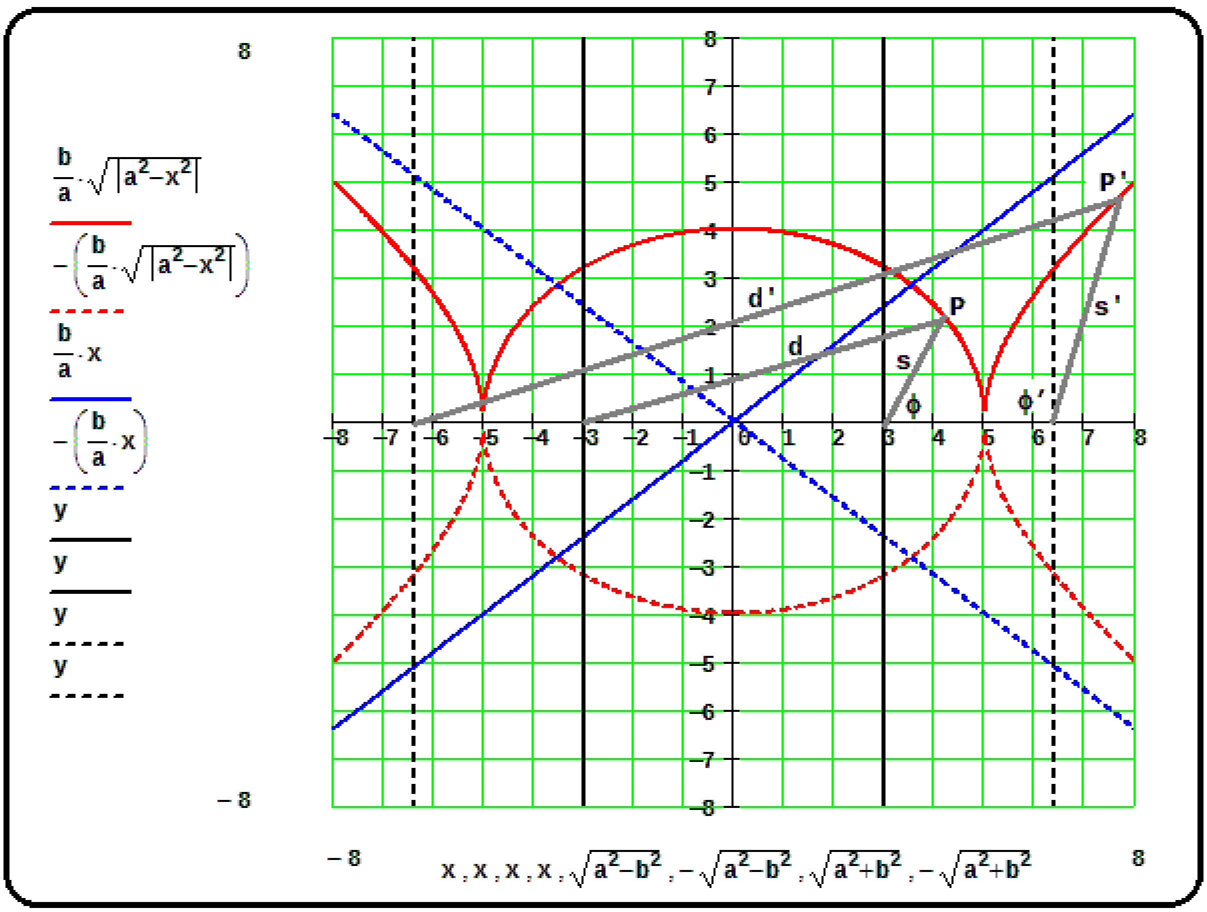
[ϕ]	[x]	[y]	[day]	[s]	[sol]
0.0000	1.0000	0.0000	-0.0001	0.9833	1.4200
0.5236	0.8701	0.4927	29.4763	0.9855	1.4138
1.0472	0.5124	0.8586	59.2025	0.9914	1.3968
1.5708	0.0167	0.9997	89.3686	0.9997	1.3738
2.0944	-0.4874	0.8731	120.0548	1.0081	1.3509
2.6180	-0.8618	0.5072	151.2019	1.0144	1.3343
3.1416	-1.0000	0.0000	182.6199	1.0167	1.3283
3.6652	-0.8618	-0.5072	213.0530	1.0144	1.3343
4.1888	-0.4874	-0.8731	243.4898	1.0081	1.3509
4.7124	0.0167	-0.9997	273.9301	0.9997	1.3738
5.2360	0.5124	-0.8586	304.3702	0.9914	1.3968
5.7596	0.8701	-0.4927	334.8067	0.9855	1.4138
6.2832	1.0000	0.0000	365.2398	0.9833	1.4200

This table was prepared using the preceding protocols.

The Hyperbola

Imagine 2 points on a plane. Imagine that a line from each point is drawn to a common third point. In an ellipse the sum of the length of the two lines will always be equal to the length of the major axis [2a]. In a hyperbola the difference in the length of the two lines will always be equal to the length of the major axis [2a]. This concludes the discussion of the ellipse. Now let us move on to the discussion of the hyperbola.

This illustration depicts both the ellipse and the hyperbola as the same general function. The prime foci are both situated on the right. The variable [s] for the ellipse and the variable [s′] for the hyperbola represent the respective distances from their respective prime foci to a respective point on the ellipse or hyperbola. The variable [d] and the variable [d′] represent the respective distances from the respective opposite foci. The variable [ϕ] represents the angle between the prime focus and a point on the ellipse from the right. The variable [ϕ′] represents the angle between the prime focus and a point on the hyperbola from the left.



For the Ellipse: d = 2a – s

For the Hyperbola: d′ = 2a + s′

21: $c = \sqrt{a^2 + b^2}$

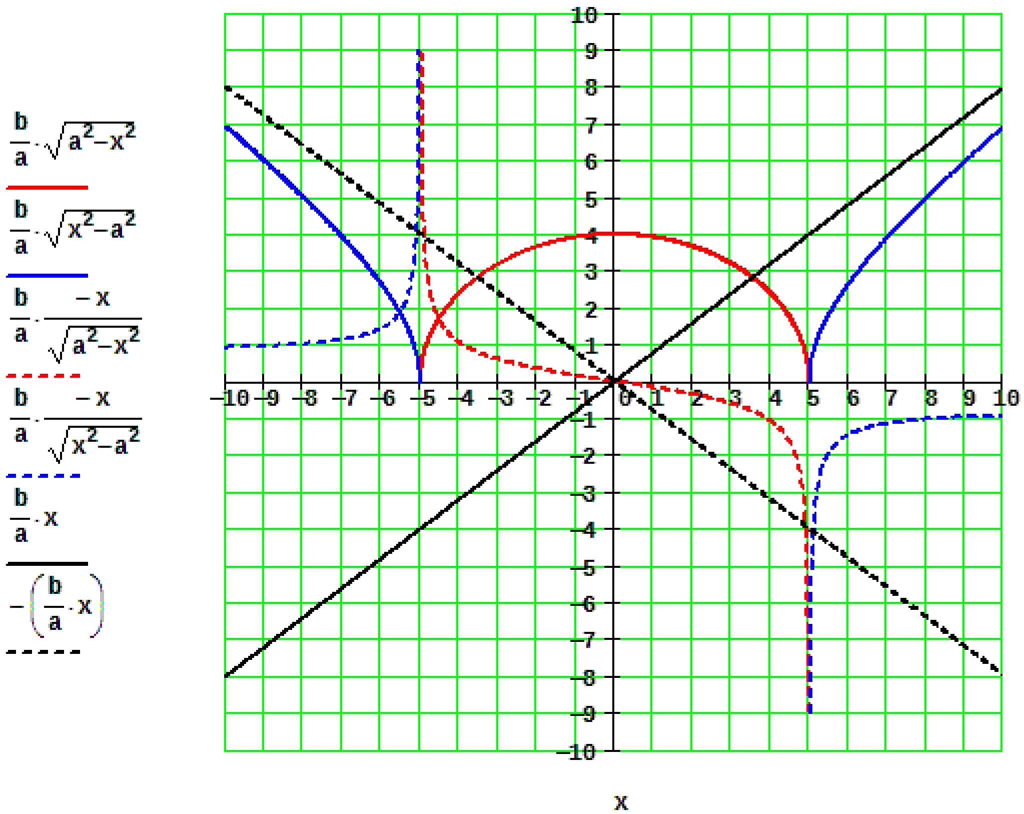
22: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ $\qquad b^2 \cdot x^2 - a^2 \cdot y^2 = a^2 \cdot b^2$ $\qquad a^2 \cdot y^2 = b^2 \cdot x^2 - a^2 \cdot b^2$

23: $y = \pm \dfrac{b}{a} \cdot \sqrt{a^2 - x^2}$ $\qquad y^2 = \dfrac{b^2 \cdot x^2 - a^2 \cdot b^2}{a^2}$ $\qquad y^2 = \dfrac{b^2 \cdot \left(x^2 - a^2\right)}{a^2}$

24: $\dfrac{d}{dx}\left(\dfrac{b}{a} \cdot \sqrt{x^2 - a^2}\right) = \dfrac{-b \cdot x}{a \cdot \sqrt{x^2 - a^2}}$

25: $\dfrac{d}{dx}\left[-\left(\dfrac{b}{a} \cdot \sqrt{x^2 - a^2}\right)\right] = \dfrac{b \cdot x}{a \cdot \sqrt{x^2 - a^2}}$

a = 5 b = 4 c = 6.403



Examine the preceding illustration. It provides the most basic coordinate equations for the hyperbola. The only significant difference is that the square root of [$a^2 - x^2$] for an ellipse has been replaced with the negative of the expression as the square of [$x^2 - a^2$]. [c] has been pushed out past [a]. An example is included. Observe that the ellipse is included for comparison. In the example, only the details are shown for when [$y \geq 0$]. When [$y < 0$], the negative [-] of the displayed function must be taken.

Here is an itemized description of these most basic formulae and equations for the hyperbola.

21: This particular [c] represents the length between the origin on the major axis and either of the two foci on the major axis of the hyperbola. This [c] is not to be confused with the [c] from the ellipse. For the ellipse it was found by taking the square root of the difference between the respective squares of the semi-major axis [a] and the semi-minor axis [b]. For the hyperbola it was found by taking the square root of the sum of the respective squares of the semi-major axis [a] and the semi-minor axis [b].

22: Here is the general equation for a hyperbola set to [1]. The only difference between it and the general equation for an ellipse is that the second term is subtracted instead of added. The shown work is the process of isolating [y^2] on the left.

23: This represents the y-coordinate [y] as the square root of [y^2]. There are two solutions symmetrically reflected about the major axis.

24: This is the differential equation for the hyperbola for when [y] is greater than or equal to [0] [$y \geq 0$]. This differential represents the instantaneous slope of the curve of the hyperbola. The only difference between it and the comparable differential for the ellipse is that [a^2] and [x^2] under the radical have been "flipped" about the subtraction sign [-].

25: This is the differential equation for the hyperbola for when [y] is less than [0] [$y < 0$]. This differential represents the instantaneous slope of the curve of the hyperbola. The only difference between it and the comparable differential for the ellipse is that [a^2] and [x^2] under the radical have been "flipped" about the subtraction sign [-].

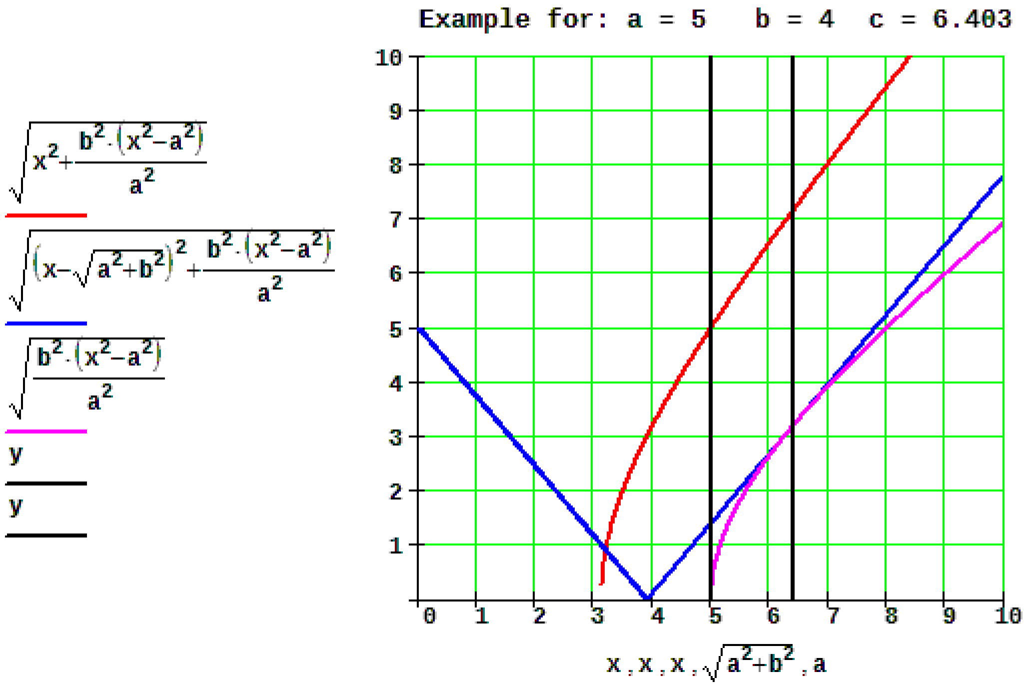
26: [b/a] represents an asymptote of the hyperbola. This is an imaginary line that the hyperbola may approach, but can never reach.

27: [-b/a] represents the other asymptote of the hyperbola. This is also an imaginary line that the hyperbola may approach, but can never reach.

28: [d] represents the distance from the opposite focus of an ellipse to a point [P] on the perimeter as a joint function of the semi-major axis [a] and the distance [s] from the prime focus to a point on the perimeter. [$d = 2a - s$].

29: [d'] represents the distance from the opposite focus of a hyperbola to a point [P'] on the perimeter as a joint function of the semi-major axis [a] and the distance [s'] from the prime focus to a point on the perimeter. [$d' = 2a + s$].

This next illustration depicts the length [s’(x)] as a function of [x] representing the distance between the prime focus of the hyperbola and a given point [P’] on the path of the hyperbola.



$$s = \sqrt{\left(x - \sqrt{a^2 + b^2}\right)^2 + \frac{b^2\cdot\left(x^2 - a^2\right)}{a^2}} \qquad c = \sqrt{a^2 + b^2} \qquad b = \sqrt{c^2 - a^2}$$

Variables [c] and [s] pertain to the hyperbola.

$$s = \sqrt{(x - c)^2 + \frac{\left(c^2 - a^2\right)\cdot\left(x^2 - a^2\right)}{a^2}}$$

$$s = \sqrt{\frac{a^2\cdot x^2 - a^2\cdot 2\cdot x\cdot c + a^2\cdot c^2 + c^2\cdot x^2 - c^2\cdot a^2 - a^2\cdot x^2 + a^4}{a^2}}$$

$$s = \sqrt{\frac{a^4 - a^2\cdot 2\cdot x\cdot c + c^2\cdot x^2}{a^2}} = \frac{a^2 - c\cdot x}{a} = a - \frac{c}{a}\cdot x \qquad \text{For: } x \geq a$$

Observe the linear progress of [s’] with respect to the x-coordinate. The equations proving this linear progression are shown. Observe that the reduced equation for [s’(x)] for the hyperbola is identical to the similar reduced equation for the ellipse. The preceding graph also depicts the length [r’] between the origin of the hyperbola and a given point [P’] on the path of the hyperbola. In addition, the two vertical lines representing the base and the focus of the hyperbola respectfully are shown.

This illustration depicts the relationships of the hyperbola between [x’], [a], [b], [c’], [s’], and [ϕ’] given [a] and [b].

30: [s’] represents the reduced linear formula for the distance from the prime focus of the hyperbola on the right to a point [P’] along the path of the hyperbola. Everything to the left of [+a,0] is irrelevant. The proven equation for the hyperbola is identical to the proven equation for ellipse, with a twist. If the absolute value is taken, then the two equations are identical. However, at the critical point the return goes negative. This is common in linear equations. The solution is to take the negative of the equation by flipping the terms on the left and right sides of the subtraction sign.

31: [ϕ’] represents the angle between the semi-major axis and a point [[P’] along the hyperbola.

32: [x’], or in this case simply [x], is provided as an inversion of [ϕ’]. This is a necessity for when observing the changes of apparent angle.

On the left side of the graph is depicted the x-coordinate as a function of [ϕ’]. This is the upper dashed black trace. Also, on the left side of the graph is the distance [s’(ϕ’)] in dashed red.

$a = 5 \quad b = 4$

$$\left(\frac{\sqrt{a^2+b^2}}{a}\cdot x - a\right)\cdot(x \ge a)$$

$$\sqrt{\frac{b^2\cdot(x^2-a^2)}{a^2}}$$

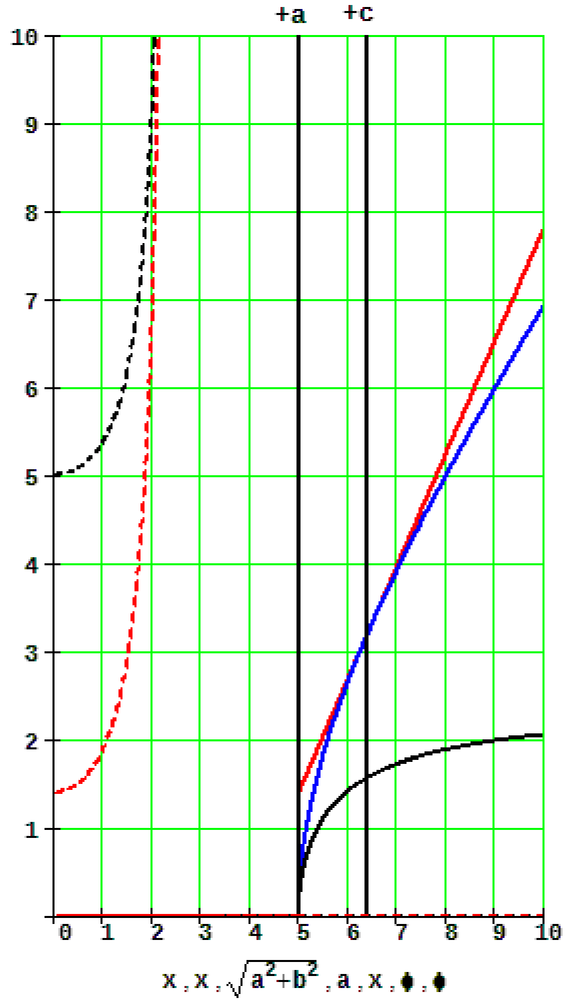
y

y

$$\pi - \mathrm{acos}\left[\frac{x-\sqrt{a^2+b^2}}{\left(\frac{\sqrt{a^2+b^2}}{a}\cdot x - a\right)}\right]$$

$$\frac{a\cdot\cos(\phi)+\sqrt{a^2+b^2}}{\left(1+\frac{\sqrt{a^2+b^2}}{a}\cdot\cos(\phi)\right)}\cdot(\phi<\pi)$$

$$\left[\frac{\sqrt{a^2+b^2}}{a}\cdot\frac{a\cdot\cos(\phi)+\sqrt{a^2+b^2}}{\left(1+\frac{\sqrt{a^2+b^2}}{a}\cdot\cos(\phi)\right)}-a\right]\cdot(\phi<\pi)$$



$$30:\quad s' = \frac{c}{a}\cdot x - a \quad \text{-------------------------------------} \quad x \ge a$$

$$31:\quad \phi' = \pi - \mathrm{acos}\left[\frac{x-c}{\left(\frac{c}{a}\cdot x - a\right)}\right] = \pi - \mathrm{acos}\left[\frac{x-\sqrt{a^2+b^2}}{\left(\frac{\sqrt{a^2+b^2}}{a}\cdot x - a\right)}\right]$$

$$32:\quad x = \frac{a\cdot\cos(\phi)+c}{1+\frac{c}{a}\cdot\cos(\phi)} = \frac{a\cdot\cos(\phi)+\sqrt{a^2+b^2}}{1+\frac{\sqrt{a^2+b^2}}{a}\cdot\cos(\phi)} \quad \text{----------} \quad -\pi < \phi \le \pi$$

Now let us consider the swept area of the Celestial hyperbola. The following formulae, equations, and expressions have been itemized and will be explained, item by item.

$$33:\ \int_0^x \left(\frac{b}{a}\cdot\sqrt{x^2-a^2}\right)dx = \frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)$$

$$34:\ K = \frac{b}{a}\cdot\left(\frac{a^2}{2}\cdot\ln(a)\right)$$

$$35:\ \int_a^x \left(\frac{b}{a}\cdot\sqrt{x^2-a^2}\right)dx = \frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)+K$$

$$36:\ L = \left(\frac{x-\sqrt{a^2+b^2}}{2}\right)\cdot\left(\frac{b}{a}\cdot\sqrt{x^2-a^2}\right)$$

$$37:\ \text{Swept Area} = \frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)+K-L$$

$$38:\ \text{Swept Time} = \frac{\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)+K-L}{a\cdot b\cdot\pi}$$

33: This is the integral equation for [y(x)] for a change of [x] representing the accumulated area under the curve of the hyperbola. It is a general indefinite integral taken from the mathematical handbook. As such it begins at [x = 0]. The trouble is that the hyperbola begins at [x = +a].

34: Uppercase [K] represents the reduced symbolic constant of the preceding integral for when [x = +a]. This solution is used to eliminate the extraneous returns from the preceding integral. At [x = +a] it has a positive value.

35: Here we have the addition of the constant [k] to the integral of the hyperbola. It has been written this way due to the constraints of the page on which it is written. At [x = +a], the value of the integral will be zero.

36: Uppercase [L] represents the area of a right triangle rooted to [+c,0] of the hyperbola. Between [+a,0] and [+c,0] it has a negative value. Past [+c,0] it has a positive value. This provides an automatic sign switch. It is subtracted from the previously adjusted integral of the hyperbola.

37: This is the final equation for the swept area of the orbital hyperbola as a function of [x]. The preceding right triangle was added when [x < +c] and subtracted when [x > +c].

38: This is the final equation for time from perihelion for the orbital hyperbola as a function of [x]. The swept area of the hyperbola is divided by the area of the comparable ellipse. The comparable ellipse has the same value for the semi-major axis [a] and the semi-minor axis [b]. The units of time for the hyperbola are in terms of years of the comparable ellipse.

Observe that there is little mention of the inversion of these formulae with regard to calculating the value of time. Many students and their professors have sought a simple solution, and have all failed. Thus it is much more convenient to prepare a detailed table and interpolate between the entries.

The following illustration depicts many of the preceding functions for the time for the orbital hyperbola.

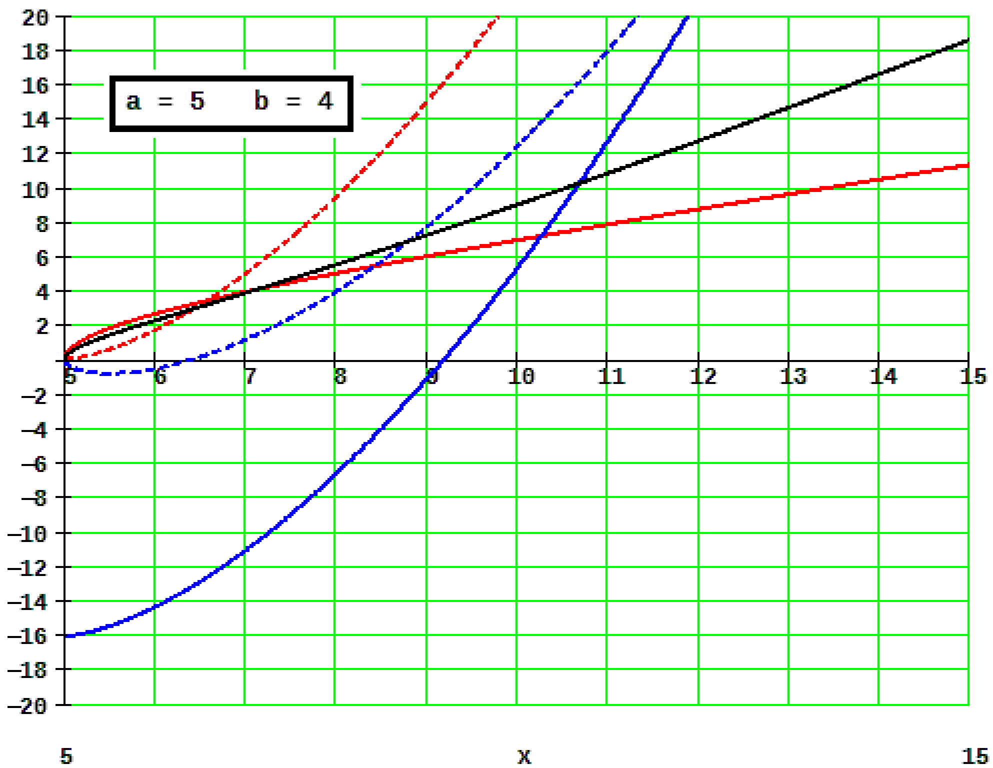
$$\frac{b}{a}\cdot\sqrt{x^2-a^2}$$

$$\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)$$

$$\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)+\frac{b}{a}\cdot\left(\frac{a^2}{2}\cdot\ln(a)\right)$$

$$\left(\frac{x-\sqrt{a^2+b^2}}{2}\right)\cdot\left(\frac{b}{a}\cdot\sqrt{x^2-a^2}\right)$$

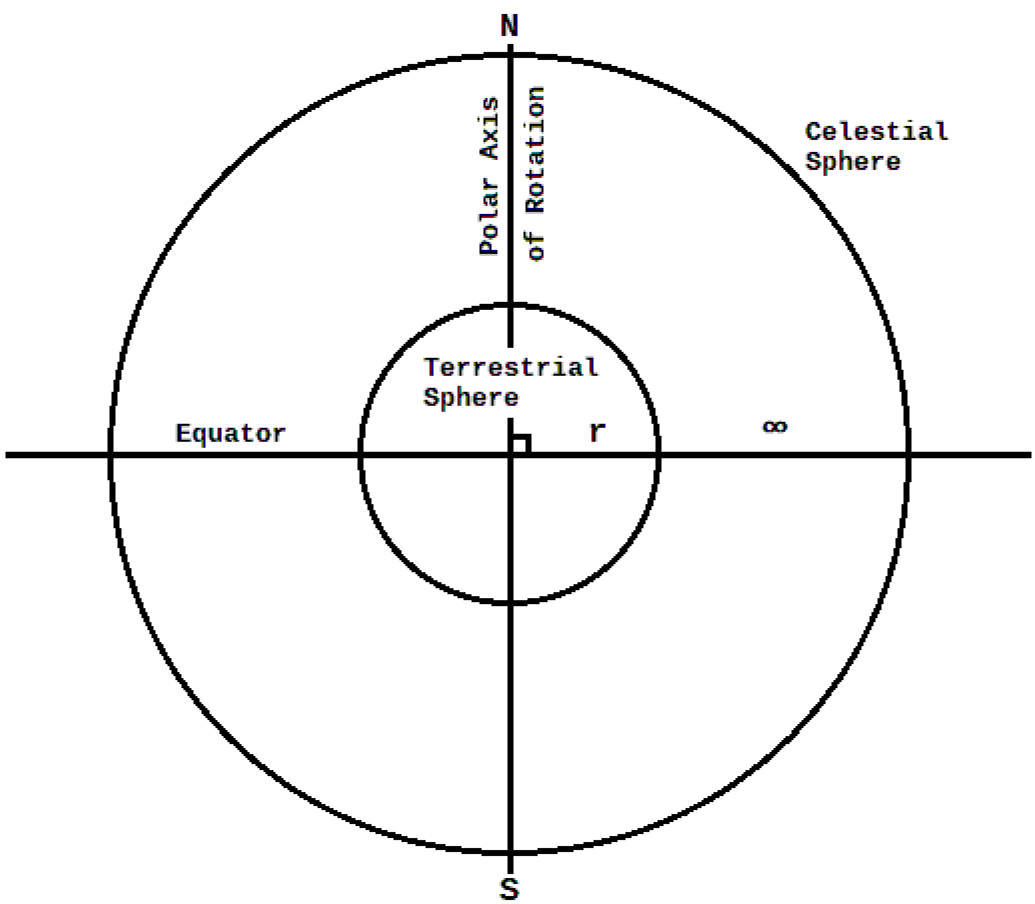
$$\frac{b}{a}\cdot\left(\frac{x\cdot\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2-a^2}\right)\right)+\frac{b}{a}\cdot\left(\frac{a^2}{2}\cdot\ln(a)\right)-\left(\frac{x-\sqrt{a^2+b^2}}{2}\right)\cdot\left(\frac{b}{a}\cdot\sqrt{x^2-a^2}\right)$$



This concludes this brief background in astrophysics for working out the solar constant.

Chapter III

An Introduction to the Terrestrial Sphere and to the Celestial Sphere with Respect to the Nomenclature



The Terrestrial Sphere

The Terrestrial Sphere is the sphere of the Earth on which we live. It is the plains, the deep valleys, the rolling hills, and the high mountains. It is the rivers, the lakes, the seas, and the great oceans. It is the green grass, the forests, the villages of men, and the farmsteads of men. It is the wild animals, the domestic animals, and the fish of the sea. The Terrestrial Sphere is our home.

The Terrestrial Sphere is of a finite radius. It rotates on a polar axis. This rotation results in two hemispheres, each with a pole and separated by an equator. Men have long divided up the Terrestrial Sphere into imaginary coordinates consisting of latitude and longitude. Let us now look at the attributes of the Terrestrial Sphere objectively.

East: The Sun rises in the East. The Sun must always rise in the East. The term “East” is defined as the cardinal direction of the rising of the Sun. Mathematically, East is frequently assigned a positive [+] value.

West: The Sun sets in the West. The Sun must always set in the West. The term “West” is defined as the cardinal direction of the setting of the Sun. Mathematically, West is frequently assigned a negative [-] value.

North: The hemisphere of the nativity of an isolated tribe of men or of a dominant culture is defined as the North Hemisphere. The term “North” is defined as the cardinal direction of the pole of the North Hemisphere. Mathematically, North is frequently assigned a positive [+] value.

South: The hemisphere opposite the hemisphere of the nativity of an isolated tribe of men or of a dominant culture is defined as the South Hemisphere. The term “South” is defined as the cardinal direction of the pole of the South Hemisphere. Mathematically, South is frequently assigned a negative [-] value.

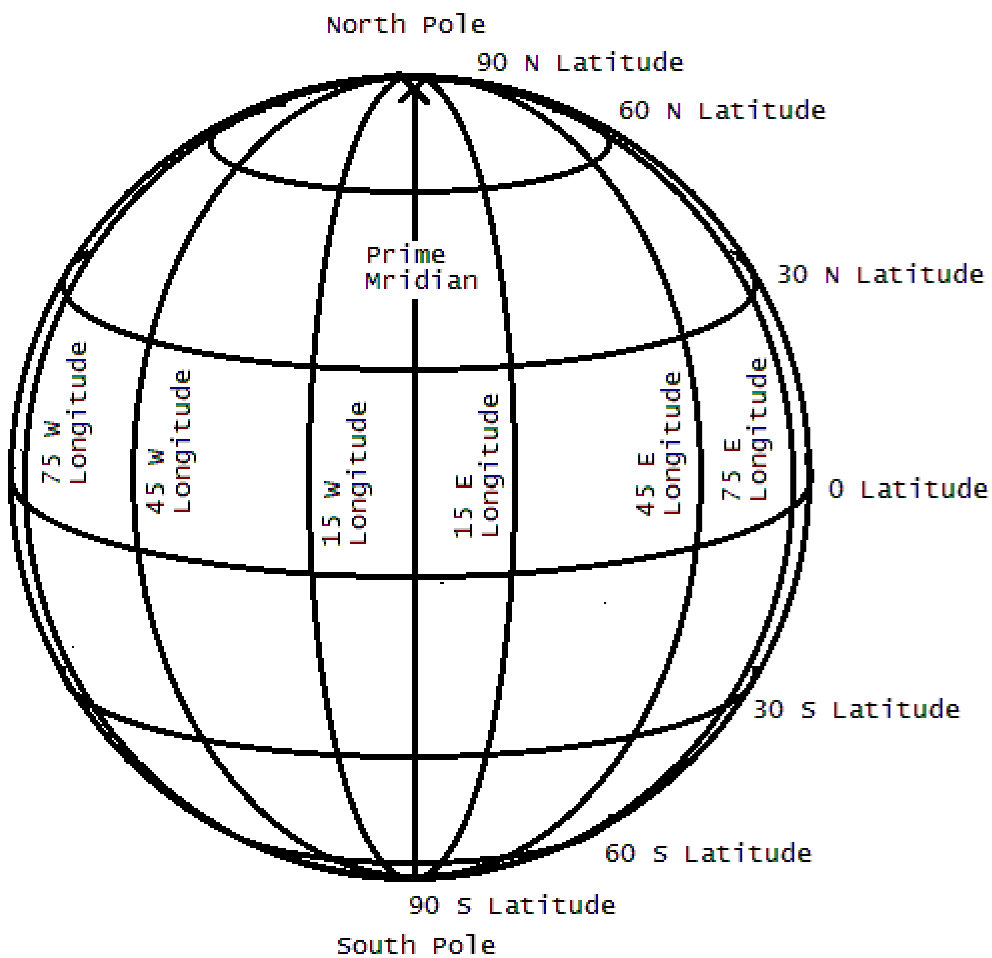
North Pole: The North Pole is the singular point in the North Hemisphere that lies over the axis of rotation of the Terrestrial Sphere.

South Pole: The South Pole is the singular point in the South Hemisphere that lies over the axis of rotation of the Terrestrial Sphere.

Equator: The Equator is the line that is equidistant from either pole of the Terrestrial Sphere.

Latitude: The coordinate system for the surface of the Terrestrial Sphere consists of imaginary lines of latitude and longitude. The lines of latitude are concentric nested parallel rings surrounding the Poles. They are frequently referred to as “parallels.” The lines of latitude are expressed as an angle subtended from the center of the Terrestrial Sphere. The equator is assigned a value of [0]. The North pole is conventionally assigned a value of 90 degrees [$+90^\circ$] or $+\pi/2$ radians [$+\pi/2$ rad].

Longitude: The coordinate system for the surface of the Terrestrial Sphere consists of imaginary lines of latitude and longitude. The lines of longitude are all imaginary circles whose radius is equal to the radius of the Terrestrial Sphere. All of the lines of longitude converge at the poles of the Terrestrial Sphere. They are commonly referred to as the meridians. There are two kinds of meridians. One is a Prime Meridian and the other is the Local Meridian. The Prime Meridian is a political assignation and the Local Meridian is a matter of practicality. The lines of longitude are commonly expressed in terms of degrees from either the Prime Meridian or the Local Meridian. A longitude East of the Prime Meridian or the Local Meridian is assigned a positive [+] value. A longitude West of the Prime Meridian or the Local Meridian is assigned a negative [-] value. The Prime Meridian or the Local Meridian is assigned a value of [0].



The Celestial Sphere

The Celestial Sphere is the sphere that surrounds the Terrestrial Sphere. The Celestial Sphere is of an infinite [∞] radius with respect to the center of the Terrestrial Sphere. The Celestial Sphere is the home of the fixed stars. The Sun, the Moon, the Planets, and the comets wander about the Celestial Sphere. All of the inhabitants of the Celestial Sphere have one thing in common; they are all out of reach of the men who dwell on the Surface of the Terrestrial Sphere. The Celestial Sphere is an imaginary sphere created by projecting the Terrestrial Sphere onto the Heavens. The Celestial Sphere is essentially a mathematical construct. Distances in the Celestial Sphere are solely measured as angles.

The Terrestrial Sphere rotates inside of the Celestial Sphere. Both have a common center. The polar axis of the Terrestrial Sphere is projected onto the Celestial Sphere as the polar axis of the Celestial Sphere. The Terrestrial Sphere rotates from West to East inside of the Celestial Sphere.

The inhabitants of the Terrestrial Sphere are unaware of their rotation inside of the Celestial Sphere. To the inhabitants of the Terrestrial Sphere, the Celestial Sphere appears to rotate from East to West. The Sun, the Moon, the Planets, the Comets, and the Stars are all carried by this apparent rotation. To the inhabitants of the Terrestrial Sphere, the apparent rotation of the Celestial Sphere is about two common points; the North Celestial Pole and the South Celestial Pole. These two Celestial Poles are but logical projections of the respective Terrestrial Poles.

Let us now take a look at some of the nomenclature when working with the Celestial Sphere.

North Celestial Pole: The North Celestial Pole is a projection of the North Terrestrial Pole onto the apparent rotating Celestial Sphere. To an inhabitant of the Terrestrial Sphere standing at the North Terrestrial Pole; the North Celestial Pole will lie directly overhead. To an inhabitant of the Terrestrial Sphere standing at the Terrestrial Equator; the North Celestial Pole will lie directly North on the level horizon. An inhabitant of the South Terrestrial Hemisphere will be unable to view the North Celestial Pole. In all cases the Celestial Sphere will appear to revolve about the Celestial poles.

South Celestial Pole: The South Celestial Pole is a projection of the South Terrestrial Pole onto the apparent rotating Celestial Sphere. To an inhabitant of the Terrestrial Sphere standing at the South Terrestrial Pole; the South Celestial Pole will lie directly overhead. To an inhabitant of the Terrestrial Sphere standing at the Terrestrial Equator; the South Celestial Pole will lie directly South on the level horizon. An inhabitant of the North Terrestrial Hemisphere will be unable to view the South Celestial Pole. In all cases the Celestial Sphere will appear to revolve about the Celestial poles.

Declination: The system of latitude of the Terrestrial Sphere, subtended from the center of the Terrestrial Sphere, is projected onto the Celestial Sphere as a Celestial latitude. The Celestial latitude in normally referred to as the declination. The Celestial Equator is assigned a declination of [0]. The Celestial North Pole is assigned a declination of [N90°], [+90°], or [+π/2]. The Celestial South Pole is assigned a declination of [S90°], [-90°], or [-π/2].

Right Ascension: The Right Ascension represents the Celestial longitude. The Celestial Sphere rotates about the Terrestrial Sphere with a period equal to a sidereal day. This creates a problem for establishing an index point. The declination of the Sun alternately moves North and South in the Celestial Sphere. The index for the Right Ascension by convention is set at the point that the ascending Sun is crossing the Celestial Equator. This corresponds to the Spring Equinox. During the course of a year the Sun completes a circle about the Celestial Sphere. There are 366.24 sidereal days in a year. There are 365.24 solar days in a year, or exactly one day less than the number of sidereal days. The length of a solar day varies by ±4 seconds. The mean solar day has by definition a length of 86,400 seconds [24 hr x 60 min x 60 sec]. The units of the Right Ascension are expressed as [hr:min:sec]. 1 hour [hr] is equal to [15º] or [π/12] radian [rad].

The Local Sphere

The Local Sphere is the apparent Celestial Sphere of an observer randomly located on the surface of the Terrestrial Sphere. The equatorial plane of the Local Sphere is tilted with respect to the equatorial plane of the Celestial Sphere.

Altitude: The altitude of the Local Sphere represents the vertical angle with respect to the level horizon of the observer on the surface of the Terrestrial Sphere. By convention the altitude is expressed in units of degrees. Above the level horizon is assigned a positive [+] value and below the level horizon is assigned a negative [-] value. The altitude has a range of [+90º,+π/2] for the point directly above the observer to [[-90º,-π/2] for the point directly below the observer.

Azimuth: The azimuth angle is the circular measure of the horizontal plane of the observer. It is indexed to the North Terrestrial Pole. By convention the azimuth is expressed in terms of positive degrees from North. It has a range [0º,0] to [360º,2π]. North is assigned a value of [0º] or [0] rad. East is assigned a value of [90º] or [π/2] rad. South is assigned a value of [180º] or [π] rad. West is assigned a value of [270º] or [3π/2] rad.

The Ecliptic Sphere

The Ecliptic Sphere is the Celestial Sphere whose equatorial base plane is the orbit of the Earth about the Sun. In the Ecliptic Sphere the Sun always circles about the Ecliptic Equator. The Ecliptic Sphere is also tilted to the Celestial Sphere.

Obliquity: The obliquity is the angle that the polar axis of the Terrestrial and Celestial Spheres make to the polar axis of the Ecliptic Sphere. The obliquity of the Terrestrial Sphere varies from 22.25º to 24.5º over a period of 41,000 years. The current obliquity is 23.45º.

Precession of the Equinoxes: The precession of the equinoxes represents a 25,800 year rotation of the polar axis of the Terrestrial Sphere about the polar axis of the Ecliptic Sphere.

North Ecliptic Pole: According to the datum for 2000 AD, the North Ecliptic Pole was located at [RA18:00:00, N66º35’38.55’] with respect to the Celestial Sphere.

Chapter IV

The Position of the Sun

for a level unobstructed plain

In order to calculate for the potential power and energy of the Sun as received by particular locale on the Earth it is necessary to know where the Sun will be at any time of the day. It is also necessary to know at what time the Sun will be at particular location in the Local Sphere. This chapter will examine the applicable formulas and their appropriate arrangements.

For the Local Sphere there are three coordinate elements. These are the time of the day, the altitude of the Sun, and the azimuth of the Sun. The formulae to be examined are also used in navigation, astronomy, and astrology.

The three coordinate elements will be rewritten as needed into a spherical trigonometry format. The angles will be rewritten as needed into a radian measure of angles.

Let us now define the variables to be used in this chapter. A logical order will be attempted.

[lat]: This variable represents the Terrestrial latitude of the reporting station expressed in terms of the conventional degrees with respect to the Terrestrial Equator. A North latitude is assigned a positive [+] value and a South latitude is assigned a negative [-] value.

[L]: This variable represents the conversion of the latitude [lat] in degrees to [L] in radians.

[L = lat x (π/180)]

[b]: This variable represents the latitude of the reporting station with respect to the Terrestrial North Pole. It is a positive value with a range of [0] at the Terrestrial North Pole to [π] at the Terrestrial South Pole. The angle is expressed in terms of radians.

[b = (π/2) – L = (π/2) – lat x (π/180)]

[dec]: This variable represents the declination of the Sun in the Celestial Sphere with respect to the Celestial Equator in terms of the conventional degrees. A North declination is assigned a positive [+] value and a South declination is assigned a negative [-] value.

[D]: This variable represents the conversion of the declination [dec] in degrees to [D] in radians.

[D = dec x (π/180)]

[c]: This variable represents the declination of the Sun with respect to the Celestial North Pole. It is a positive value with a range of [0] at the Celestial North Pole to [π] at the Celestial South Pole. The angle is expressed in terms of radians.

[c = (π/2) – D = (π/2) – dec x (π/180)]

[hra]: This variable represents the hour angle from the nadir of the Sun [midnight] in terms of positive hours. [hra] is indicated by the rotation of the Celestial Sphere.

[α]: This is variable [α], pronounced “alpha”, the first of the lowercase Greek letters. It is used here represent the angle that the Sun makes about the Celestial Polar Axis. It is measured from the zenith of the Sun [Noon]. [α] has a range of ±π radians. It is derived from [hra].

[α = hra x (π/12) - π]

[alt]: This variable represents the altitude of the Sun with respect to the level unobstructed horizon of the reporting station in terms of the conventional degrees. Altitudes above the horizon are assigned a positive [+] value and altitudes below the horizon are assigned a negative [-] value.

[a]: This variable represents the altitude of the Sun with respect to a point directly above the reporting station in terms of radians. It is always positive.

[a = (π/2) – alt x (π/180)]

[azi]: This variable represents the positive azimuth of the Sun with respect to the Terrestrial North Pole. By convention it is expressed in terms of degrees. North [N] is assigned a value of [0°]. East [E] is assigned a value of [90°]. South [S] is assigned a value of [180°]. West [W] is assigned a value of [270°].

[γ]: This is angle [γ], pronounced “gamma”, the third of the lowercase Greek letters. It is used here to represent the azimuth of the Sun with respect to the Terrestrial North Pole in terms of radians.

[γ = azi x (π/180)]

Calculating the Altitude of the Sun as a function of the Time of the day.

Let us begin by considering the calculations for determining the altitude of the Sun with respect to the level horizon of the reporting station.

Given: Terrestrial Latitude of Reporting Station in Degrees as [lat].
Celestial Declination of Sun in degrees as [dec].
Hour Angle from Nadir of Sun in Hours as [hra].

lat = given   dec = given   hra = independent

$$b = \frac{\pi}{2} - lat \cdot \frac{\pi}{180} \qquad c = \frac{\pi}{2} - dec \cdot \frac{\pi}{180} \qquad \alpha = hra \cdot \frac{\pi}{12} - \pi$$

$$\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$$

$$a = \text{acos}(\cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha))$$

$$alt = \left(\frac{\pi}{2} - a\right) \cdot \frac{180}{\pi}$$

There are two trigonometric identities to consider. These are as follows: **[sin(u) = (π/2) – cos(u)]** and **[cos(u) = (π/2) – sin(u)]**. When these two identities are employed for the preceding formulae we get:

[sin(alt) = sin(lat) x sin(dec) + cos(lat) x cos(dec) x sin(hra)]

This is a simple formula for conventional work, but it gives little information of its source. These simple formulae often have problems when intermixing them. It is always best to go to the source.

Here is a practical example of the preceding equations using real parameters. There are two graphic forms shown. One is for the angles as radians from noon on radians and the other is for hours from midnight on degrees. Observe the use of the intermediary variables [p] and [q]. Since they both represent constants in the equations, they may both be pre-filled to reduce clutter in the equations.

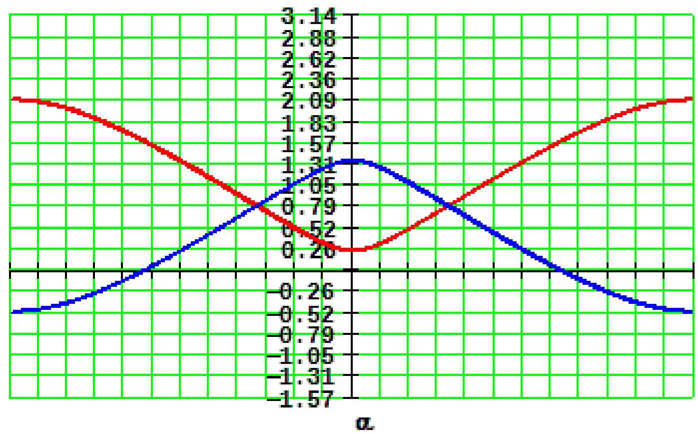
Given: $lat := 36.5$ $dec := 23.45$ Event: Summer Solstice

$$b = \frac{\pi}{2} - lat \cdot \frac{\pi}{180} = 0.934 \qquad c = \frac{\pi}{2} - dec \cdot \frac{\pi}{180} = 1.162$$

$$p := \cos(b) \cdot \cos(c) \qquad q := \sin(b) \cdot \sin(c)$$

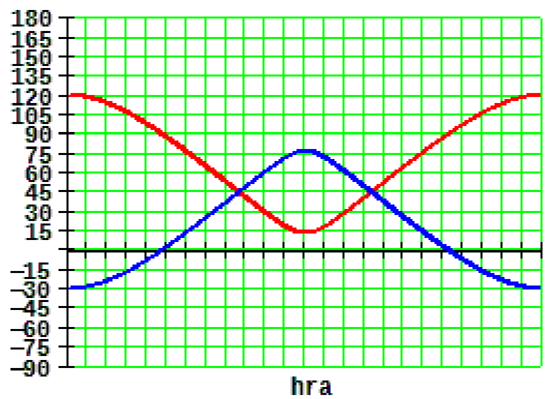
$\operatorname{acos}(p + q \cdot \cos(\alpha))$ (red)

$\frac{\pi}{2} - \operatorname{acos}(p + q \cdot \cos(\alpha))$ (blue)



$\operatorname{acos}\left(p + q \cdot \cos\left(hra \cdot \frac{\pi}{12} - \pi\right)\right) \cdot \frac{180}{\pi}$ (red)

$\left(\frac{\pi}{2} - \operatorname{acos}\left(p + q \cdot \cos\left(hra \cdot \frac{\pi}{12} - \pi\right)\right)\right) \cdot \frac{180}{\pi}$ (blue)



There are two special cases for calculating the altitude of the Sun as a function of the time of day. These are are the calculations for nadir (midnight) and zenith (Noon). In both of these cases the numerical value is [α] negated,

Case 1 for $\alpha = \pi$ or $\alpha = -\pi$: $\cos(\alpha) = -1$ [hra = 00/24 (Midnight)]

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)(-1)$$

$$\cos(a) = \cos(b)\cdot\cos(c) - \sin(b)\cdot\sin(c)$$

$$a = \operatorname{acos}(\cos(b)\cdot\cos(c) - \sin(b)\cdot\sin(c))$$

$$p := \cos(b)\cdot\cos(c) \quad q := \sin(b)\cdot\sin(c)$$

$$a = \operatorname{acos}(p - q)$$

$$alt = \left(\frac{\pi}{2} - a\right)\cdot\frac{180}{\pi}$$

Case 2 for $\alpha = 0$: $\cos(\alpha) = +1$ [hra = 12 (Noon)]

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)(1)$$

$$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)$$

$$a = \operatorname{acos}(\cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c))$$

$$p := \cos(b)\cdot\cos(c) \quad q := \sin(b)\cdot\sin(c)$$

$$a = \operatorname{acos}(p + q)$$

$$alt = \left(\frac{\pi}{2} - a\right)\cdot\frac{180}{\pi}$$

Calculating the Altitude of the Sun as a function of the Azimuth of the Sun.

The preceding calculations employed **Formula I-1a** from Chapter I pertaining to spherical trigonometry. This is the simplest and the most basic application of the law of cosines. This next series of calculations will employ **Formula I-3b** from Chapter I pertaining to spherical trigonometry. This formula is a quadratic equation.

The resulting quadratic equation for cos(γ) yields two results. Both results are correct and applicable, howbeit not at the same time.

The two returns when laid over one-another show not one, but two possible continuous traces. Both traces are correct. One trace has an origin of [γ = 0] and the other trace has an origin of [γ = π]. We are concerned with the former origin of [γ = 0].

When [γ = (π/2)] or [γ = (3π/2)], 90° or 270° respectively, the value under the radical goes to zero. When that happens, everything to the right of the [±] sign also goes to zero. This leaves us with the two special cases for 90° or 270° respectively.

$$\cos(a) = \frac{-(\cos(b)\cdot\cos(c))}{\cos(b)^2\cdot\cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2} \qquad \gamma = \frac{\pi}{2} \qquad \gamma = \frac{3\pi}{2}$$

$$alt = \frac{\pi}{2} - acos\left[\frac{-(\cos(b)\cdot\cos(c))}{\cos(b)^2\cdot\cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2}\right] \qquad \gamma = \frac{\pi}{2} \qquad \gamma = \frac{3\pi}{2}$$

This next illustration depicts the quadratic equation, its applications, and a graphic example.

I-3b: $A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

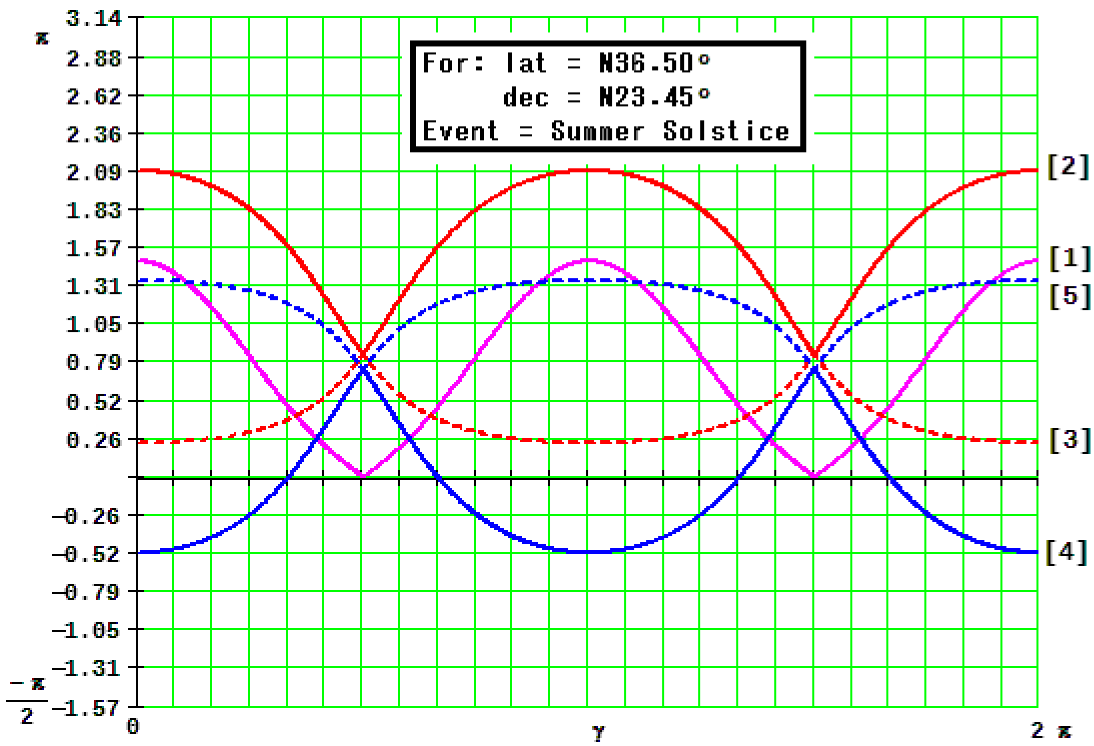
$B = 2 \cdot \cos(b) \cdot \cos(c)$

$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \gamma < \frac{\pi}{2} \qquad \gamma > \frac{3 \cdot \pi}{2}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}$$

Example:
- [1]: Value under radical.
- [2]: (+) Solution for [a].
- [3]: (-) Solution for [a].
- [4]: (+) Solution for [alt].
- [5]: (-) Solution for [alt].



Calculating the Azimuth of the Sun as a function of the Time of the Day.

This is a simple insertion of the arc-cosine of **[I-1a]** into **[a]** in **[I-3d]**. Because both [α] and [γ] have an unlimited range, there is only one minor complication. The illustration depicts the equations, the minor complication, the complication resolution, and an example.

I-1a: $$\cos(a) = \cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha)$$

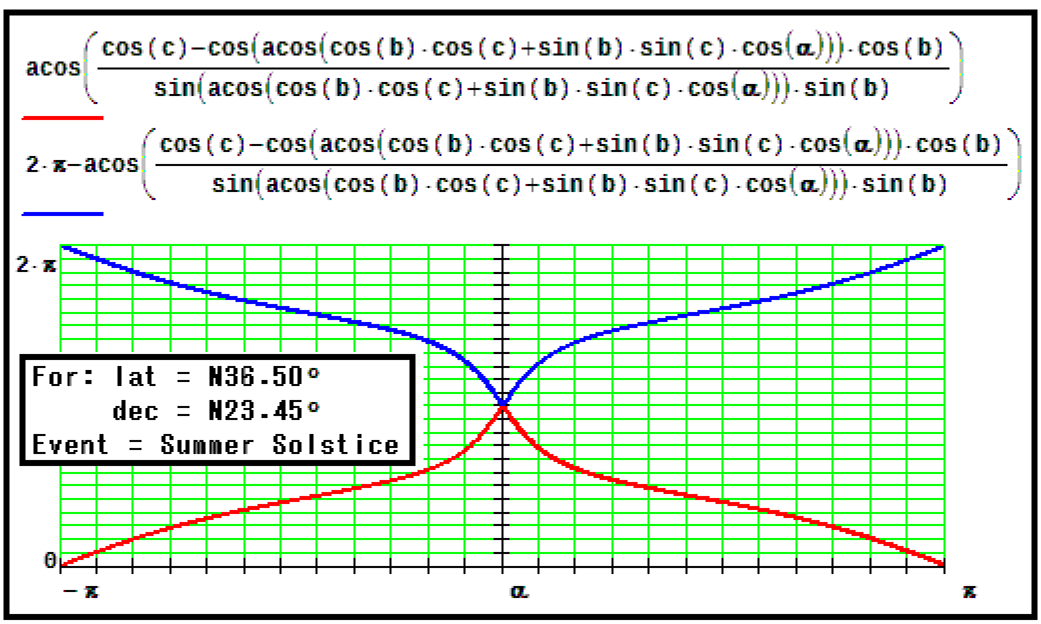
$$a = \operatorname{acos}(\cos(b)\cdot\cos(c) + \sin(b)\cdot\sin(c)\cdot\cos(\alpha))$$

$$\Bigg\}\quad \alpha = hra - \pi$$

I-3d: $$\cos(\gamma) = \frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)} \qquad a \neq 0 \quad b \neq 0$$

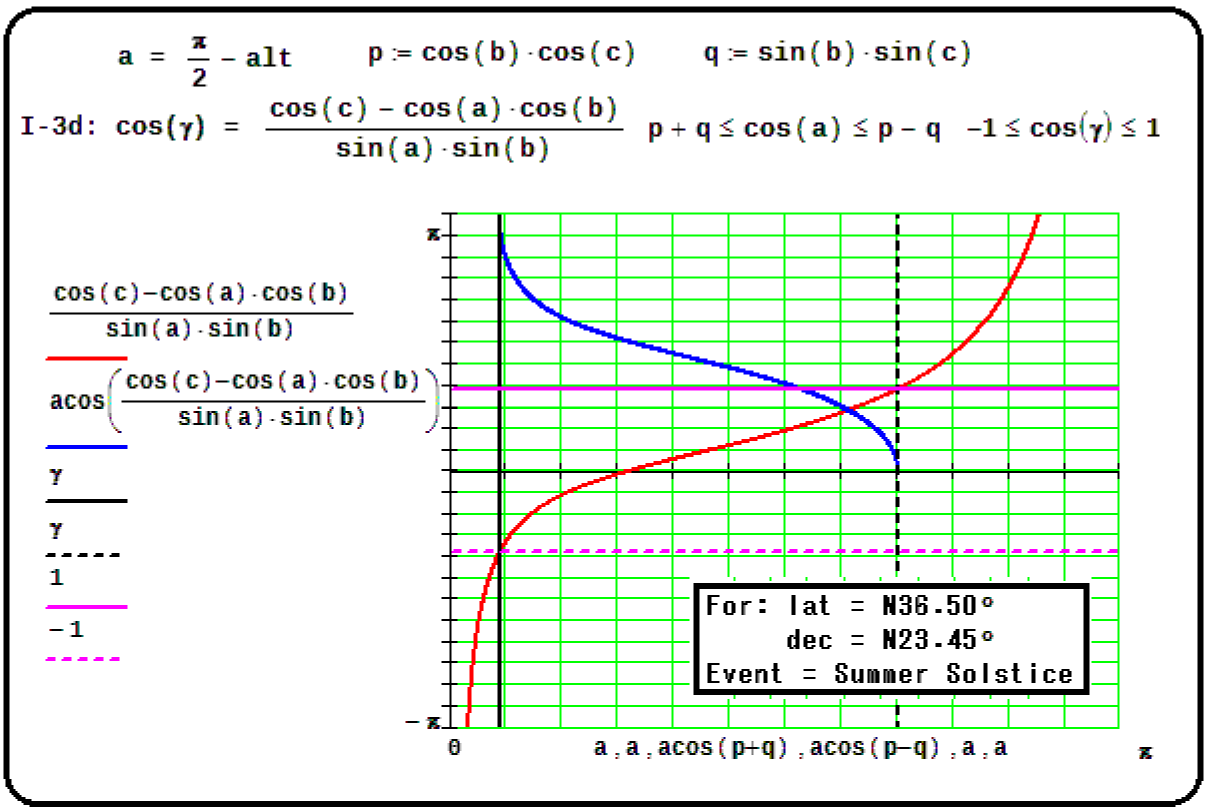
$$\gamma = \operatorname{acos}\left(\frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)}\right) \qquad a \neq 0 \quad b \neq 0 \quad \alpha \le 0$$

$$\gamma = 2\cdot\pi - \operatorname{acos}\left(\frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)}\right) \qquad a \neq 0 \quad b \neq 0 \quad \alpha > 0$$



Calculating the Azimuth of the Sun as a function of the Altitude of the Sun.

Calculating the azimuth of the Sun as a function of the altitude of the Sun employs a simple singular formula. However, there are two complications. The first complication is the range of the altitude. The other complication is the range of the return. Shown here in the illustration are the applicable formulae with the applicable conditions and limitations.



In the illustration, the solid magenta line represents the [+1] limitation of [cos(γ)].

In the illustration, the dashed magenta line represents the [-1] limitation of [cos(γ)].

In the illustration, the solid black line represents the minimum value for [a]. Alternately, this represents the maximum value for the altitude by the formula **[alt(max) = (π/2) – (p + q)]**.

In the illustration, the dashed black line represents the maximum value for [a]. Alternately, this represents the minimum value for the altitude by the formula **$[alt(max) = (\pi/2) - (p - q)]$**.

In the illustration, the red trace represents the value of [cos(γ)] as a function of [a].

Observe how the four straight lines "box in" the red trace of [cos(γ)]. This "boxed" area defines the limitations of the function.

The blue trace represents the simple calculated value of [γ]. Observe how the entire return for the function is constrained between the lower and upper limits for [a], i.e. the solid and the dashed black lines.

This is a simple function that returns a [±] azimuth with respect to the Terrestrial North Pole. The formula as given and the graph has shown only depict the [+] value. To obtain the positive [-] value; no, this is not a contradictory statement; the [-] value must be added to [2π] as **$[\gamma' = 2\pi - \gamma]$**.

The use of standardized "clock" time is good for maintaining schedules, but has little to do with the natural world. A surveyor operates in the natural world under the Sun. The surveyor cannot not depend on a standardized clock when "shooting" the Sun to obtain a base line. The surveyor must use a system where the natural time by the Sun, or any other Celestial object, is automatically incorporated. The formulae for calculating the azimuth of the Sun as a function of altitude as well as the altitude of the Sun as a function of the azimuth automatically incorporates the natural time. All the surveyor has to do is to do the calculations, adjust the altitude of the transit, and wait until he or she can get the Sun in the cross-hairs of the scope. The resulting azimuth will then be the pre-calculated azimuth.

Calculating the Time of the Day as a function of the Altitude of the Sun

It is sometimes necessary to know the time that the Sun, or any other Celestial object, will be at a particular altitude. Most often this is the Local Solar Time(s) of sunrise and Sunset for when the Sun is lying on the level horizon. The given formula returns one result, but with two applications. One application is before zenith and the other application is after zenith.

Under certain conditions, most often in the polar regions, the Sun never intercepts the horizon. This is the case of the land of the midnight Sun, or alternately, the midnight shadow. The latter may also occur in deep valleys in lower latitudes. When no intercept happens, the cosine of the function goes out of bounds and no return is possible. Yet, a return is required. This conundrum is resolved by assigning a time of “sunrise” or “sunset.”

If the Sun never sets the time of “sunrise” is assigned a value of **[hra = 0]** or **[α = -π]**. This is a practical arrangement for a species that lives under the Sun. Under these circumstances, the time of the nadir of the Sun will act as the time of “sunrise.” In addition, the time of “sunset” will be assigned a value of **[hra = 2π]** or **[α = π]**.

If the Sun never rises the time of “sunrise” is assigned a value of **[hra = π]** or **[α = 0]**. This is also a practical arrangement for a species that lives under the Sun. Under these circumstances, the time of the zenith of the Sun will act as the time of “sunrise.” In addition, the time of “sunset” will be assigned a value of **[hra = π]** or **[α = 0]**. Here, both “sunrise” and “sunset” are identical.

The equations, formulae, expressions, and an example are displayed in the following illustration.

The Red trace represents the value of [cos(a)].

The solid and dashed black traces represents the limits of [cos(a)].

The blue trace represents the normal operation of the function. The value shown for the blue trace represents the positive time displacement from the Solar Noon.

Not shown, but indicated, is the conversion of [α] from zenith to [hra] from nadir.

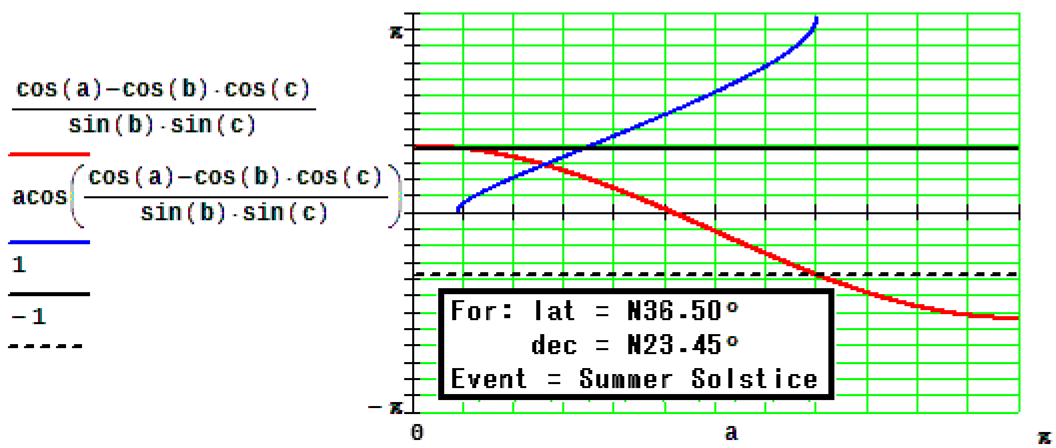
All of the angles, both for altitude and time, are expressed in terms of radians. **[1 rad = (180°/π) = (12hr/π)]**

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$\text{I-1d:}\quad \cos(\alpha) = \frac{\cos(a) - \cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)} \qquad b \neq 0 \qquad c \neq 0 \qquad -1 \le \cos(\alpha) \le 1$$

IF	$\cos(\alpha) < -1$	THEN	$\alpha = \pi$	[Assigned Value]
IF	$-1 \le \cos(\alpha) \le 1$	THEN	$\alpha = \operatorname{acos}(\cos(\alpha))$	[Calculated Value]
IF	$\cos(\alpha) > 1$	THEN	$\alpha = 0$	[Assigned Value]

$hra(AM) = \pi - \alpha \qquad hra(PM) = \pi + \alpha \qquad$ [2 Returns]



Calculating the Time of the Day
as a function of the Azimuth of the Sun

The particular series of calculations for determining the time of the day as a function of the azimuth of the Sun given the constants of the latitude and the declination is a bit of a monster. It is accomplished by inserting the quadratic cosine of the altitude of the Sun as a function of the azimuth of the Sun into the same cosine placeholder in the formula for the time of the day as a function of the altitude of the Sun. There are four discreet solutions evenly separated by an independent variable of (π/2). For the following solution all angles including time are expressed as radians.

I-3b: $A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

$B = 2 \cdot \cos(b) \cdot \cos(c)$

$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \gamma < \frac{\pi}{2} \qquad \gamma > \frac{3 \cdot \pi}{2}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}$$

Substitute right hand side of return for cos(a) from I-3b for cos(a) placeholder in I-1d

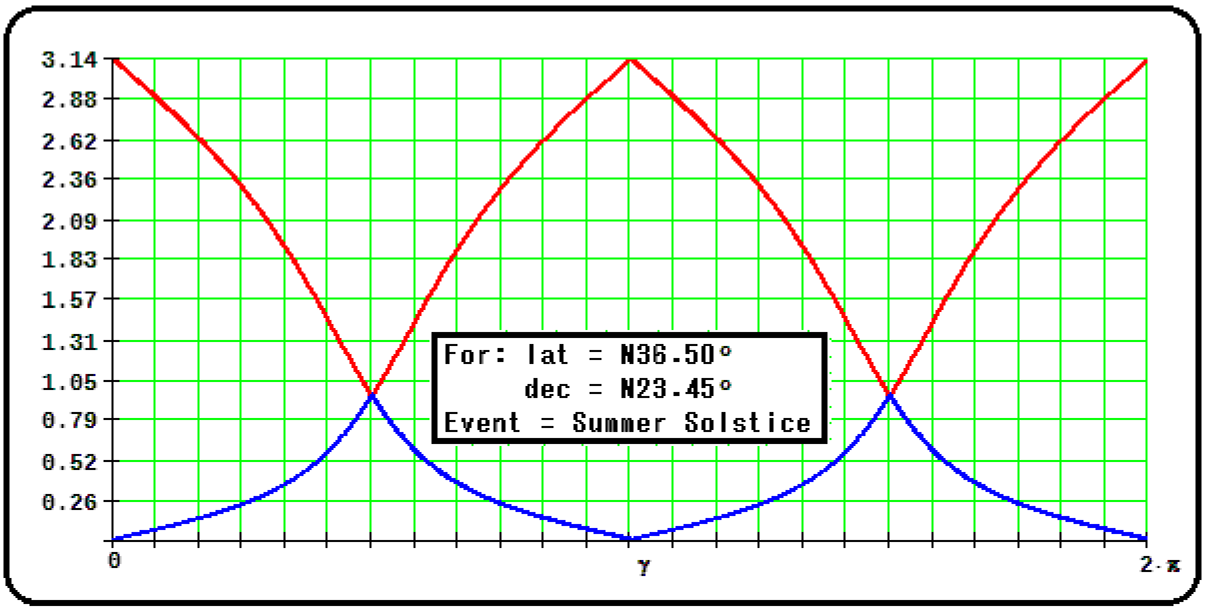
I-1d: $$\cos(\alpha) = \frac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}$$

$$\alpha = \operatorname{acos}(\cos(\alpha)) \begin{cases} hra = \pi - \alpha & \gamma \le \pi \\ hra = \pi + \alpha & \gamma > \pi \end{cases}$$

This process results in 4 solutions:
All are correct within their parameters.

This procedure was the only way to eliminate any extraneous [a] variables.

This illustration depicts the positive value of variable [α] as a function of [γ] representing the positive azimuth of the Sun from North. Observe that the traces are discreetly divided into four sections with discontinuities at [γ = 0], [γ = (π/2), [γ = π], [γ = (3π/2)], [γ = 2π]. Observe that there are two traces. The red trace represents the situation where the radical of the quadratic has a [+] value and the blue trace represents the situation where the radical of the quadratic has a [-] value.



Let us now consider the sections one at a time.

The first section from [γ = 0] to [γ = (π/2)] has the blue trace rising from [α = 0] to [α = max]. Here [α] is assigned a negative [-] value.

The second section from [γ = (π/2)] to [γ = π)] has the red trace rising from [α = min] to [α = π]. Here [α] is assigned a negative [-] value.

The third section from [γ = π)] to [γ = (3π/2)] has the red trace descending from [α = π] to [α = min]. Here [α] is assigned a positive [+] value.

The fourth section from [γ = (3π/2)]] to [γ = (2π] has the red trace descending from [α = max] to [α = 0]. Here [α] is assigned a positive [+] value.

Summary of the Formulae

Here is a summation of the preceding formulae without the graphic illustrations and the detailed explanations. Those were only needed to help in understanding the source of the formulae.

Given the Terrestrial latitude of the site and the Celestial declination of the Sun as immutable constants, there are three mutable elements. The three mutable elements are the time of the day, the altitude of the Sun, and the azimuth of the Sun. These three mutable elements have six pairs of relationships. All six of these relationships are given here.

By convention, altitude is expressed as degrees with respect to the level horizon of the site. The range of the altitude is [-90º] to [+90º]. 1 degree is equal to [π/180] radians.

By convention, latitude is expressed as degrees with respect to the Terrestrial Equator. The range of the latitude is [-90º] to [+90º]. 1 degree is equal to [π/180] radians.

By convention, time is expressed as positive hours with respect to the solar midnight. The range of the latitude is [00 hr] to [24 hr]. 1 hour is equal to [π/12] radians.

1: alt(hra):

$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad \alpha = hra - \pi$

I-1a: $\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$

$a = acos(\cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha))$

$alt = \frac{\pi}{2} - a$

2: alt(azi):

$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad \gamma = azi$

I-3b: $A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

$B = 2 \cdot \cos(b) \cdot \cos(c)$

$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \gamma < \frac{\pi}{2} \qquad \gamma > \frac{3 \cdot \pi}{2}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}$$

$a = \mathrm{acos}(\cos(a)) \qquad alt = \frac{\pi}{2} - a$

3: azi(hra):

$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad \alpha = hra - \pi$

I-1a: $\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$

$a = \mathrm{acos}(\cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha))$

} Substitute solved [a] from [1a] into [a] placeholders in [3d].

I-3d: $\cos(\gamma) = \frac{\cos(c) - \cos(a) \cdot \cos(b)}{\sin(a) \cdot \sin(b)} \qquad a \ne 0 \quad b \ne 0 \quad -1 \le \cos(\gamma) \le 1$

$$\gamma = \mathrm{acos}\left(\frac{\cos(c) - \cos(a) \cdot \cos(b)}{\sin(a) \cdot \sin(b)}\right) \quad \begin{cases} a \ne 0 \quad b \ne 0 \quad \alpha \le 0 \\ -1 \le \cos(\gamma) \le 1 \end{cases}$$

$$\gamma = 2 \cdot \pi - \mathrm{acos}\left(\frac{\cos(c) - \cos(a) \cdot \cos(b)}{\sin(a) \cdot \sin(b)}\right) \quad \begin{cases} a \ne 0 \quad b \ne 0 \quad \alpha > 0 \\ -1 \le \cos(\gamma) \le 1 \end{cases}$$

$azi = \gamma$

4: azi(alt):

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

I-3d: $$\cos(\gamma) = \frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)} \qquad a \neq 0 \qquad b \neq 0 \qquad -1 \le \cos(\gamma) \le 1$$

IF $\cos(\gamma) < -1$ THEN $\gamma = \pi$ [Assigned Value]

IF $-1 \le \cos(\gamma) \le 1$ THEN $\gamma = \operatorname{acos}(\cos(\gamma))$ [Calculated Value]

IF $\cos(\gamma) > 1$ THEN $\gamma = 0$ [Assigned Value]

$azi(AM) = \gamma$ $azi(PM) = 2\cdot\pi - \gamma$ [2 Returns]

5: hra(alt):

$$a = \frac{\pi}{2} - alt \qquad b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

I-1d: $$\cos(\alpha) = \frac{\cos(a) - \cos(b)\cdot\cos(c)}{\sin(b)\cdot\sin(c)} \qquad b \neq 0 \qquad c \neq 0 \qquad -1 \le \cos(\alpha) \le 1$$

IF $\cos(\alpha) < -1$ THEN $\alpha = \pi$ [Assigned Value]

IF $-1 \le \cos(\alpha) \le 1$ THEN $\alpha = \operatorname{acos}(\cos(\alpha))$ [Calculated Value]

IF $\cos(\alpha) > 1$ THEN $\alpha = 0$ [Assigned Value]

$hra(AM) = \pi - \alpha$ $hra(PM) = \pi + \alpha$ [2 Returns]

6: hra(azi):

$$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad \gamma = azi$$

I-3b: $$A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$$

$$B = 2 \cdot \cos(b) \cdot \cos(c)$$

$$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \gamma < \frac{\pi}{2} \qquad \gamma > \frac{3 \cdot \pi}{2}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}$$

Substitute right hand side of return for cos(a) from I-3b for cos(a) placeholder in I-1d

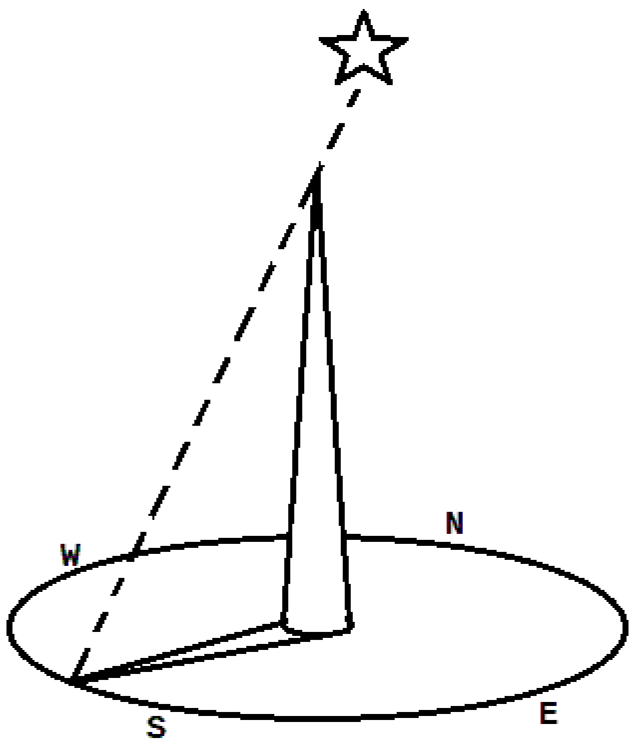
I-1d: $$\cos(\alpha) = \frac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}$$

$$\alpha = \operatorname{acos}(\cos(\alpha)) \begin{cases} hra = \pi - \alpha & \gamma \le \pi \\ hra = \pi + \alpha & \gamma > \pi \end{cases}$$

These positional formulae are useful in predicting the solar power and solar energy of a site. They are also useful in navigation, surveying, and civil engineering.

Chapter V

The Path of the Shadow of the Sun for a level unobstructed plain



Sometimes it is interesting or useful to know the path of the tip of the shadow of the Sun as it passes over a level unobstructed surface. This chapter is dedicated to providing the essential formulae for doing so.

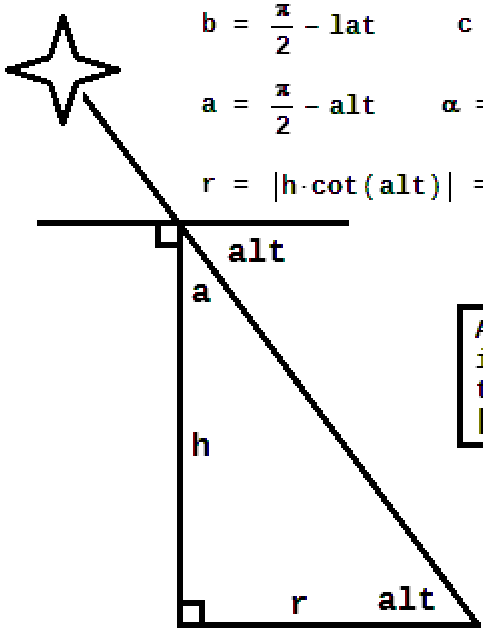
For these arguments an [x-y] coordinate system will be used. North will be assigned a [+y] value. East will be assigned a [+x] value. South will be assigned a [-y] value. West will be assigned a [-x] value.

This illustration depicts the basic geometry and formulae for calculating the length of the radius of the Shadow on the level Earth as a function of both the time of the day and the azimuth of the Sun.

$$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec$$

$$a = \frac{\pi}{2} - alt \qquad \alpha = hra - \pi \qquad \gamma = azi$$

$$r = |h \cdot \cot(alt)| = |h \cdot \tan(a)| \quad a < \frac{\pi}{2}$$



A negative value of [r] is not possible. Thus, the absolute value of [r] is employed.

I-1a: $\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$

I-3b: $A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

$B = 2 \cdot \cos(b) \cdot \cos(c)$

$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$

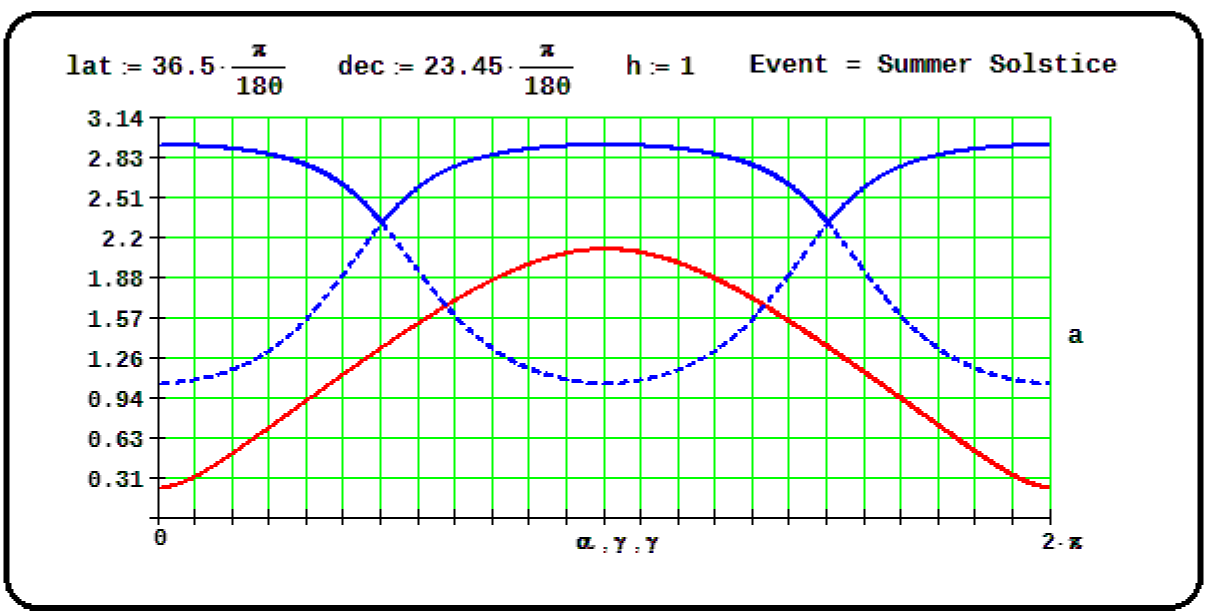
$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \gamma < \frac{\pi}{2} \qquad \gamma > \frac{3 \cdot \pi}{2}$$

$$a = \operatorname{acos}(\cos(a)) \qquad alt = \frac{\pi}{2} - a$$

The calculations for the value of [a] representing the altitude of the Sun as a function of angle [α] with respect to a point perpendicular to the level Earth directly above the site are straight-forward. The only complication is that the formula is a function of angle [α] representing the time of the zenith of the Sun. This is removed from the conventional system which begins the day cycle at the nadir of the Sun by [π] radians (12 hours). This is shown as the simple red trace in the illustrated example.

For calculating the value of [a] of the Sun as a function of angle [γ] representing the azimuth of the Sun with respect to the Terrestrial North Pole is more complicated. There are two solutions. Both solutions are correct, howbeit not simultaneously. The two solutions are linked and alternately flow in and out of one another. Decisions must me made for which sections are applicable to the specific concerns. The example shows both traces as blue. The positive [-B + radical] solution is depicted as the solid blue trace. The negative [-B – radical] solution is depicted as the dashed blue trace.

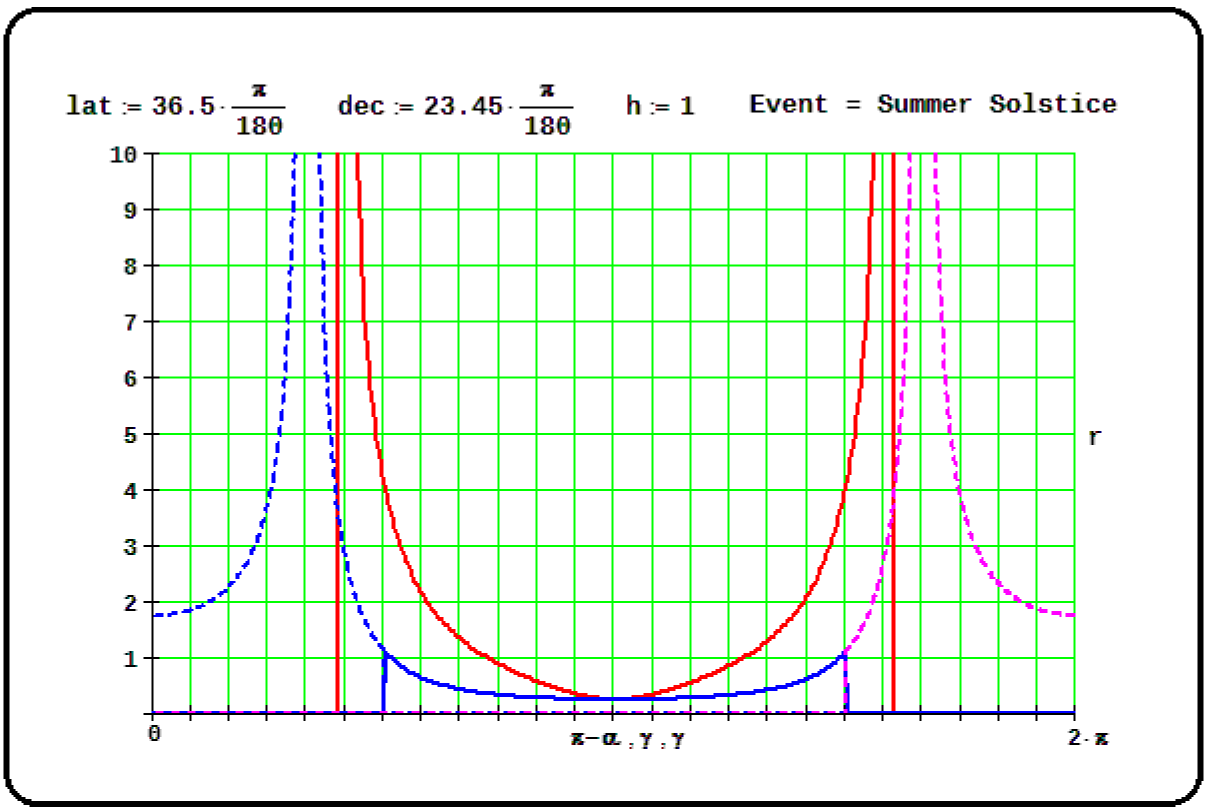


This next illustration depicts an example of the radial length of the shadow of the Sun using the given formulations at the beginning of this chapter. Both [r{α)] and [r(γ)] are shown.

The curved solid red trace represents [r{π-α)]. This is a displacement of [α] to represent the time of day with respect to the nadir of the Sun.

The vertical solid red trace represents the times of the day that the Sun crosses the level unobstructed horizon.

The dashed blue trace, the solid blue trace, and the dashed magenta trace; They all represent the applicable solutions of [r(γ)]. They are only valid within the confines of the vertical red lines.



x,y(γ)

Once the length of the shadow of the Sun is established, the next part is to break in into its x-coordinate and its y-coordinate. For the arguments here; the x-coordinate will represent the East-West component and the y-coordinate will represent the North-South component. Angle [γ] represents the positive azimuth of the Sun with respect to the Terrestrial North Pole. Angle [γ] is expressed here as radians. The x-coordinate utilizes the cosine of [γ] [cos(γ)] and the y-coordinate utilizes the sine of [γ] [sin(γ)].

I-3b: $$A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$$

$$B = 2 \cdot \cos(b) \cdot \cos(c)$$

$$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

$$a = \operatorname{acos}(\cos(a)) \qquad r = h \cdot \tan(a)$$

$$x = r \cdot \cos(\gamma) \qquad y = r \cdot \sin(\gamma)$$

Judgment and reason must be applied when using these formulae. There are two solutions for the x-coordinate and there are two solutions for the y-coordinate. All are correct, but alternate through one-another. This next illustration depicts the unqualified results of these formulae.

$lat := 36.5 \cdot \frac{\pi}{180}$ $dec := 23.45 \cdot \frac{\pi}{180}$ $h := 1$ Event = Summer Solstice

$b := \frac{\pi}{2} - lat$ $c := \frac{\pi}{2} - dec$

I-3b: $A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2$

$B = 2 \cdot \cos(b) \cdot \cos(c)$

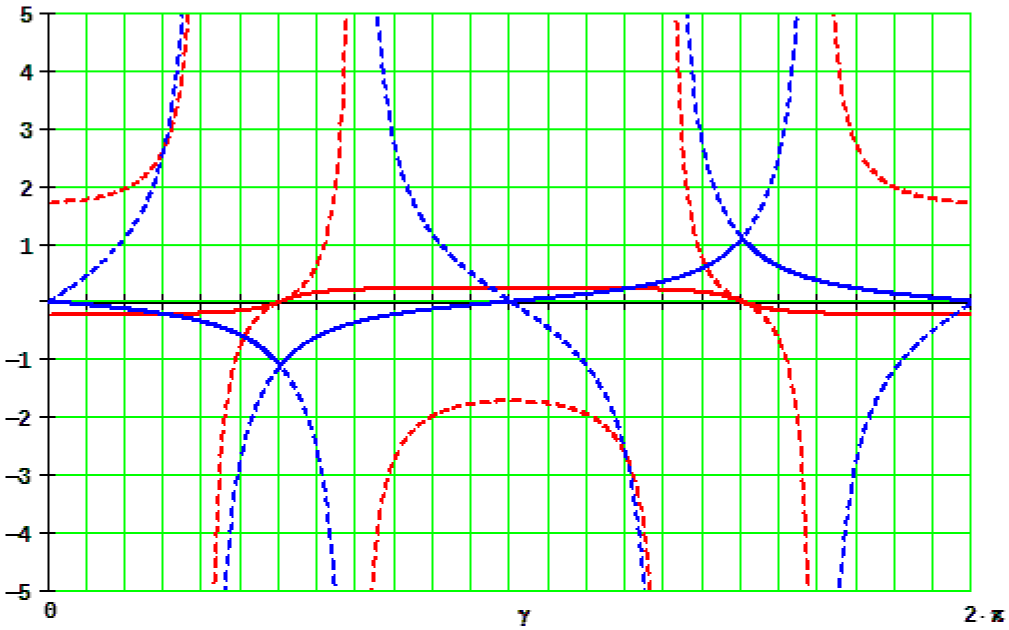
$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$

$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$ {Solid}

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$ {Dashed}

$a = \operatorname{acos}(\cos(a))$ $r = h \cdot \tan(a)$

$x = r \cdot \sin(\gamma)$ {Blue} $y = r \cdot \cos(\gamma)$ {Red}



Here is a “cleaned up” version of the preceding illustration.

$$A = \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(\gamma)^2 - \cos(b)^2 \qquad \beta := \operatorname{acos}\left(\frac{\cos(c)}{\sin(b)}\right)$$

$$B = 2 \cdot \cos(b) \cdot \cos(c) \qquad \delta := 2 \cdot \pi - \beta$$

$$C = \cos(\gamma)^2 - \cos(b)^2 \cdot \cos(\gamma)^2 - \cos(c)^2$$

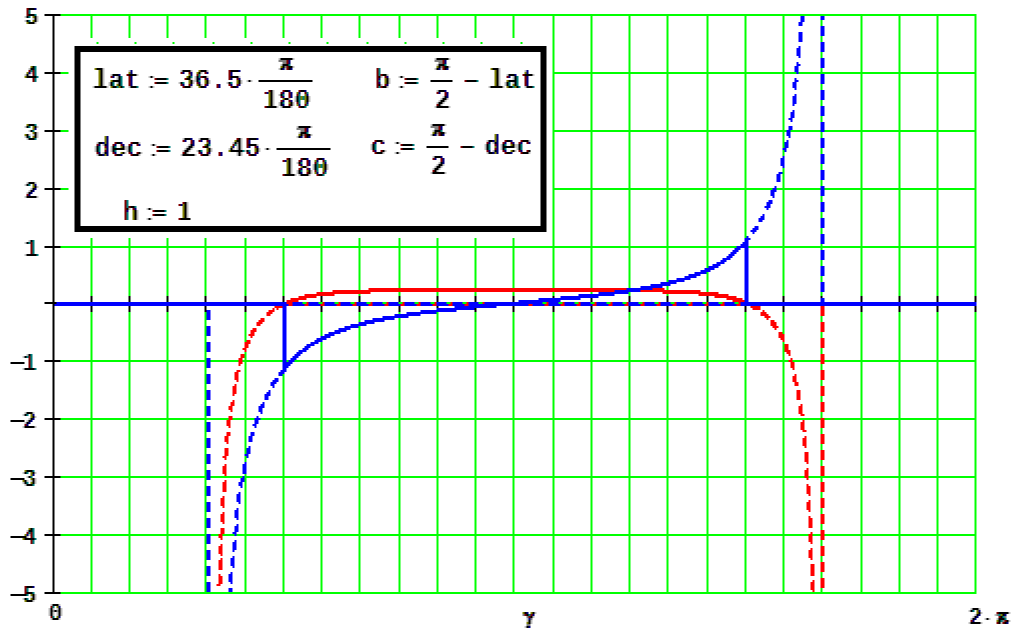
$$\cos(a) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad \left[\left(\frac{\pi}{2} \le \gamma \le \frac{3 \cdot \pi}{2}\right) \wedge (\beta \le \gamma \le \delta)\right] \qquad \{\text{Solid}\}$$

$$\cos(a) = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad \left[\left(\gamma < \frac{\pi}{2} \vee \gamma > \frac{3 \cdot \pi}{2}\right) \wedge (\beta \le \gamma \le \delta)\right] \qquad \{\text{Dashed}\}$$

$$a = \operatorname{acos}(\cos(a)) \qquad r = h \cdot \tan(a) = h \cdot \tan\left(\operatorname{acos}\left(\frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}\right)\right)$$

$$y = r \cdot \cos(\gamma) = h \cdot \cos(\gamma) \cdot \tan\left(\operatorname{acos}\left(\frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}\right)\right) \qquad \{\text{Red}\}$$

$$x = r \cdot \sin(\gamma) = h \cdot \cos(\gamma) \cdot \tan\left(\operatorname{acos}\left(\frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}\right)\right) \qquad \{\text{Blue}\}$$



Here are the explanations for the preceding illustration:

The upper left uppercase (A,B,C) represents the three elements of the quadratic form for [a(δ)].

The variable [β] in the upper right corner represents the lower usable limit of [γ] representing the azimuth angle of the Sun. This is where the Sun first rises above the level horizon. The Greek letter [β] is not required as an angle in the spherical trigonometric equations for this study, so it is free for other purposes. It is used here because it immediately preceded the Greek letter [γ]. The associated expression for angle [β] is a reduction of the formula for [γ(a)] when [a = (π/2)]. This angle represents a positive deviation from North.

The variable [δ] in the upper right corner represents the upper usable limit of [γ] representing the azimuth angle of the Sun. This is where the Sun finally sets below the level horizon. Normally, the use of the Greek letter [δ] is reserved for differential equations in calculus, but in this situation is is used simply because it is the fourth letter of the Greek alphabet immediately after [γ]. This is a positive value obtained by subtracting the angle [β] from [2π].

The two standard quadratic solution for [cos(a)] are given. To the immediate right of the two solutions are given the operable parameters with respect to angle [γ]. These are Boolean operators.

[m < n]: Condition [m] is less than n.

[m ≤ n]: Condition [m] is less than or equal to n.

[m > n]: Condition [m] is greater than n.

[m ≥ n]: Condition [m] is greater than or equal to n.

[m ∧ n]: Condition both [m] AND [n] are true.

[m ∨ n]: Condition either [m] OR [n] are true.

{Solid}: The positive quadratic solution in the example is indicated by a solid trace.

{Dashed}: The negative quadratic solution in the example is indicated by a dashed trace.

[a]: [a] represents the arc-cosine of the selected quadratic solution for [cos(a)].

[h]: [h] represents the measured height of a pole obstructing the light of the Sun.

[r]: [r] represents the total length of the shadow of the Sun with respect to [h].

[y]: [y] represents the North-South component of the shadow of the Sun. In the example it is shaded in {Red}.

[x]: [x] represents the East-West component of the shadow of the Sun. In the example it is shaded in {Blue}.

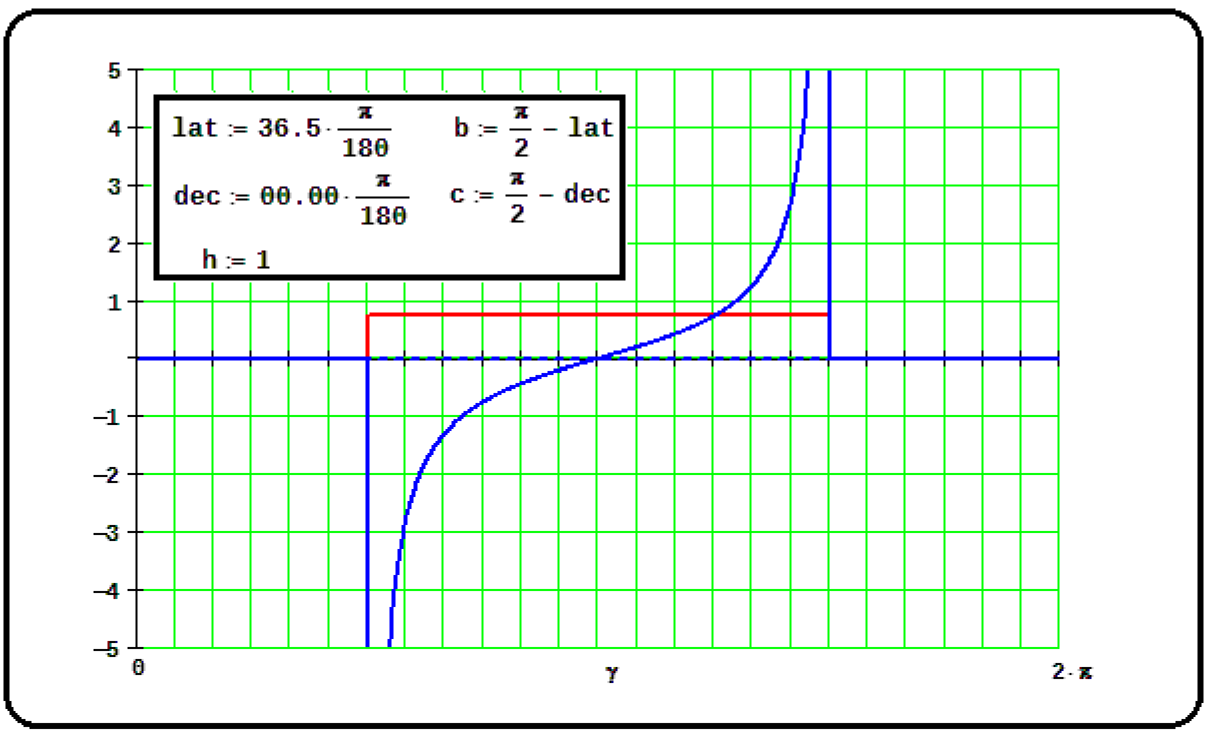
The bottom of the illustration depicts an example with the given parameters embedded. The original angles are in conventional degrees with respect to the respective Terrestrial and Celestial Equators, which have been converted to radians for use with the spherical trigonometric calculations.

A Special Case

There is a special case for the path of the shadow of the Sun. It only occurs on two days of the year. More specifically, at the times of the Vernal Equinox and the Autumnal Equinox when the declination of the Sun is on the Celestial Equator, one ascending and one descending respectfully.

On the day of the Vernal Equinox or the Day of the Autumnal Equinox, the day and the night are of equal lengths of time. The Sun rises due East and sets due West. The North-South coordinate of the shadow of the Sun is a constant. From sunrise to sunset on level ground the shadow of the Sun follow a straight path.

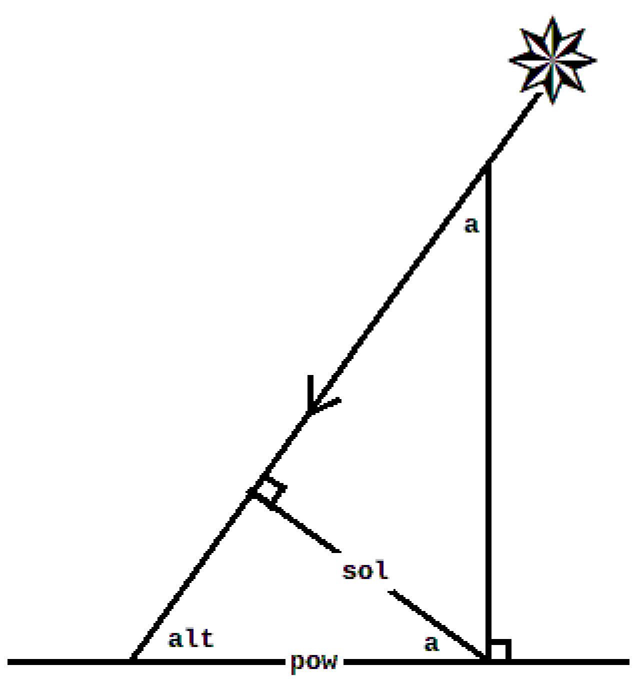
This next illustration depicts a “cleaned up” example using the same Terrestrial latitude as the previous example, but with the Celestial declination altered to [dec = 0] or [c = (π/2)]. Observe the straight line representing the North-South coordinate.



Chapter VI

Calculating for the Power & Energy of the Sun as Received on a Level Unobstructed Plain

Chapter IV introduced the calculations pertaining to the altitude, the azimuth, and the times of the Sun. The Terrestrial latitude of the site and the Celestial declination of the Sun were given as the constant parameters. This chapter will build onto the content of chapter IV by introducing the solar constant into the calculations as a third constant parameter.



The Solar Constant

As indicated in Chapter II, the solar constant varies with the position of the Earth in its elliptical orbit about the Sun. The solar constant is not truly constant.

At perihelion, i.e. the closest point to the Sun, the solar constant has a nominal value of 1.420 kilowatts per square meter. Alternately, at aphelion, i.e. the farthest point from the Sun, the solar constant has a nominal value of 1.328 kilowatts per square meter. The accepted mean nominal value for the solar constant is 1.373 kilowatts per square meter. These nominal values are predictable using simple two-body calculations.

The nominal solar constant with its elliptical variations will also vary with the health of the Sun and the position of the other planets. Due to the complexity of the equations and an insufficient database, these aspects of the solar constant are not predictable.

We must content ourselves and accept the nominal variations in the solar constant. We must accept and expect that our calculations for the solar constant can and will never be precise.

The Obliquity of the Earth

The obliquity of the Earth is the inclination of the polar axis of the Earth with respect to the polar axis of the ecliptic plane, i.e. the plane circumscribed by the orbit of the Earth about the Sun. As the Earth orbits about the Sun, the obliquity causes the declination of the Sun in the Celestial Sphere to alternate between [+23.45º] (+0.409 rad) and [-23.45º] (-0.409 rad). The obliquity of the Earth is [23.45º] (0.409 rad). With respect to the Vernal Equinox, the declination of the Sun is calculated as follows:

$$dec = \sin(yra) \cdot 0.409$$

Argument of Periapsis

The Argument of Periapsis represents the angular difference between the Vernal Equinox and the periapsis of the orbit of the Earth with respect to the Sun. Due to the 25,800 year precession of the Equinoxes about the Ecliptic Polar Axis, the Argument of Periapsis is always changing. The change is around [1º] (0.01745 rad] every [71.67] years.

The current Argument of Periapsis for the reference year of 2,000 AD is [114.2º] (1.9928 rad}. This given figure appears to be for the Earth with respect to the Sun, and does not quite agree with the observed reality. However, if the view of the situation is reversed, the complement of [75.8º] (1.3227 rad) may be applied. This complement will represent 76.9 mean solar days. This agrees with the observed reality. Thus, in the Northern Hemisphere, the Earth is closest to the Sun 76.9 days before the Vernal Equinox. Thus, on the 3rd day of February, the Earth is closest to the Sun. On that day the Solar Constant is at its greatest.

Units of Measure

This chapter on calculating the received power and energy of the Sun will utilize certain units of measure. A definition of those units of measure are in order.

[Radian (rad)]: A radian is an angle that measures the arc-length of a circle with a radius of [1]. The circumference of a circle is calculated as [cir = 2πr]. If [r = 1], then there are [2π] radians in a circle. The radian is the natural angle of angular measure and is required in all of the complicated trigonometric equations

[Degree (º)]: The degree is a conventional angular measure of men from antiquity. 360º complete a circle. The origins of the degree may be viewed in two ways.

The first way represents the days of the Sun. 360 days is only 5.24 days short of the 365.24 days of the year. There was a calendar system of antiquity that employed a civil year of 360 days. There were 12 months of exactly 30 days each. The remaining days were reserved for an annual free-for-all.

The second way represents the break-down of the number [360]. The number [360] may be broken down as [360 = 6 · 60]. Imagine that a circle is drawn using a compass. Now using the same setting on the compass, [6] evenly spaced marks may be drawn on the circumference by "walking" the compass. The number 60 may be broken down into [3·4·5]. These are the proportions of the 3-4-5 right triangle from antiquity.

The first way and the second way are connected. [3+4+5=12], the number of months in the aforementioned civil calendar. If the aforementioned civil calendar had [5] weeks per [30] day month, then there would be [60] weeks in a [12] month civil year with each week comprising [6] days. As an added bonus, the [5.24] days left over would high round up to [6] days or [1] week.

This is the convention for the degree an an angular measure. It is a cultural convenience. However, the calculations will all use the radian measure of angles for the mathematical processing.

$$\pi \text{ rad} = 180^\circ$$

[Hour Angle (hra)]: The rotating Celestial Sphere that carries the Sun, the Moon, the planets, the comets, and the stars is divided by convention into a "clock-face" with 24 divisions. Each division represents an hour. The origin of this system may be demonstrated by first dividing a circle into [6] parts using a compass and a straight-edge. Then each of the [8] parts are likewise bisected twice. The hour measure should be converted to radians for the mathematical processing.

$$\pi \text{ rad} = 12 \text{ hr}$$

[Year Angle (yra)]: The Sun appears to rotate about the Celestial Sphere with respect to the ecliptic polar axis. The ecliptic longitudinal angle that the Sun makes with respect to the background of the fixed stars is the year angle. By convention this angle is given in degrees. For mathematical processing radians are required.

[Second (s)]: There are 24 hours in a mean solar day. There are 60 minutes in an hour. There 60 seconds in a minute. There are 86,400 seconds in a mean solar day. This is the definition of the mean second. The qualifying word here is “mean.” Due to the elliptical orbit of the Earth the mean solar day varies by about ±4 seconds. Thus, the mean second will vary by about ±0.00463%. This is a cyclic variation.

[Meter (m)]: The original definition meter of the meter was 1/10,000,000th of the arc-length on the surface of the Earth from the North Terrestrial Pole to the Terrestrial Equator. Curiously enough, the half-period of a pendulum at sea-level with a center of gravity of [1] meter will be [1.003] seconds. Take your pick.

[Kilogram (kg)]: The original definition of the kilogram was 1/1,000th of a mass of water contained in a volume of [1] cubic meter.

[Newton (N)]: The Newton is a unit of force. It represents the product of the rate of acceleration expressed as [m/s^2] and the mass expressed as [kg]. The force of the weight due to gravity is expressed in terms of Newtons. The sea-level rate of acceleration due to gravity on the Earth is [9.807 m/s^2]. A 1 kg mass resting on the surface of the Earth has a "weight" of [9.807 N]. The units are [$(kg \cdot m)/s^2$].

[Joule (J)]: The Joule is a unit of potential energy. It is calculated as the product of the force in Newtons and the distance or height in meters. The units of the Joule are [$(kg \cdot m^2)/s^2$].

[Watt (W)]: A rate of released energy of one Joule per second is defined as a Watt. The units of the Watt are [$(kg \cdot m^2)/s^3$]

[Kilowatt (kw)]: The kilowatt is 1,000 Watts.

[Kilowatt-hour (kw-hr)]: The kw-hr is a standard unit of energy. An energy flow of [1 kw] per hour is defined as a [kw-hr]. The units of the [kw-hr] are [$(kg \cdot m^2)/s^2$].

This has been a general listing of the essential definitions for working with solar power and solar energy for this book. Another Book may use different units defined differently, but the basic applications will remain the same.

Calculating for Solar Power

Declaration of Employed Variables:

[sol]: The solar constant for the time of the year. The mean solar constant is 1.373 kw/m^2. It Varies from 1.328 kw/m^2 to 1.420 kw/m^2. This is a given constant.

[lat]: The latitude for these equations is expressed in terms of radians with respect to the Terrestrial Equator. The range is from [π/2] representing the Terrestrial North Pole to [-π/2] representing the South Terrestrial Pole. This is a given Constant.

[dec]: The declination for these equations is expressed in terms of radians with respect to the Celestial Equator. The range is from [π/2] representing the Celestial North Pole to [-π/2] representing the South Celestial Pole. This is a given constant.

[obs]: The minimum altitude angle of an obstacle rising above a level plain that blocks the light of the Sun. For a level unobstructed plain; [obs = 0]. This angle is expressed in terms of radians. This is a given constant.

[hra]: The hour angle is expressed in terms of positive radians from nadir (midnight) to the following nadir (midnight). The range is from [0] to [2π]. This is the independent variable.

[pow]: The power of the Sun as received on a level plain is expressed as [kw/m^2].

[a]: The calculated positive altitude of the Sun with respect to a point directly above the site. This angle is expressed as radians. It has a range of [0] to [π].

[b]: The calculated positive latitude of the site with respect to the North Terrestrial Pole. This angle is expressed as radians. It has a range of [0] to [π].

[c]: The calculated positive declination of the Sun with respect to the North Celestial Pole. This angle is expressed as radians. It has a range of [0] to [π].

[d]: The calculated positive minimum altitude of an obstacle rising above a level plain that blocks the light of the Sun. This angle is expressed as radians. It has a range of [0] to [π].

[α]: The adjusted time of day with respect to the zenith of the Sun. This angle is expressed as radians. It has a range of [-π] to [+π].

[p]: This is a convenient calculated constant representing the product of the cosine of [b] and the cosine of [c].

[q]: This is a convenient calculated constant representing the product of the sine of [b] and the sine of [c].

lat = Given Site Latitude. dec = Given Declination of Sun.
sol = Given Solar Constant. obs = Given Obtruction Altitude.

$$b = \frac{\pi}{2} - lat \quad c = \frac{\pi}{2} - dec \quad p = \cos(b)\cdot\cos(c) \quad q = \sin(b)\cdot\sin(c)$$

$$d = \frac{\pi}{2} - obs \quad \alpha = hra - \pi \quad a = acos(p + q\cdot\cos(\alpha)) \quad pow = sol\cdot\cos(a)$$

$$pow = sol\cdot(p + q\cdot\cos(\alpha)) \qquad a < d$$

$$pow = 0 \qquad a \geq d$$

Here is a real example of the preceding calculations for the power of the Sun as a function of the time of the day.

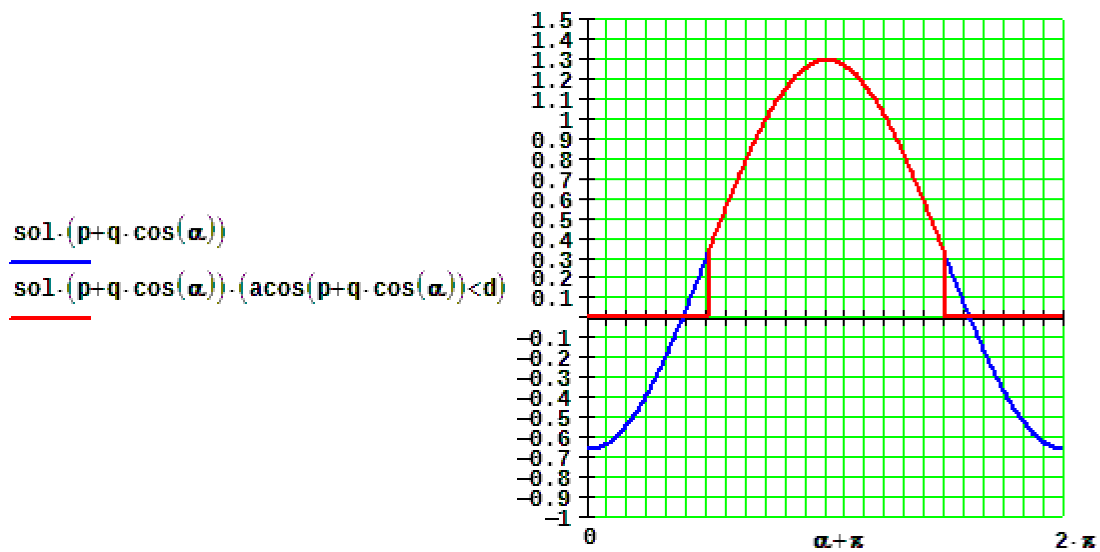
The red trace represents the power of the Sun in kilowatts per square meter with the logical restrictions in place. The orthogonal sections represent the Boolean restrictions as shown on the right side of the indicated function.

The blue trace represents the unrestricted function as a curve that continues even when the Sun is below the effective horizon. The upper part of the blue trace is masked by the upper curve of the red trace.

$lat := 36.5 \cdot \frac{\pi}{180}$ $\quad dec := 23.45 \cdot \frac{\pi}{180}$ $\quad obs := 15 \cdot \frac{\pi}{180}$ $\quad sol := 1.33$

$b := \frac{\pi}{2} - lat$ $\quad c := \frac{\pi}{2} - dec$ $\quad d := \frac{\pi}{2} - obs$

$p := \cos(b) \cdot \cos(c)$ $\quad q := \sin(b) \cdot \sin(c)$



Calculating for Solar Energy

The accumulated solar energy for the course of a day is represented by the area under the power curve. This involves taking the integral of the power formula.

There is a complication. All of the angles including time have been expressed as radians. However, the energy is expressed as kilowatt-hours where the time angle is expressed as hours. The integral equation will result in a power expression of kilowatt-radians. This needs to be multiplied by [12/π] in order to correct it to the conventional measures of time regarding power and energy.

A constant must be taken for the time of “sunrise” when the sun first becomes visible. This constant is used to set the beginning of the applicable integral to [0].

The diurnal sum of the accumulated energy occurs at the time of “sunset” when the Sun is no longer visible at the end of the day.

Declaration of Employed Variables:

[sol]: The solar constant for the time of the year. The mean solar constant is 1.373 kw/m^2. It Varies from 1.328 kw/m^2 to 1.420 kw/m^2. This is a given constant.

[lat]: The latitude for these equations is expressed in terms of radians with respect to the Terrestrial Equator. The range is from [π/2] representing the Terrestrial North Pole to [-π/2] representing the South Terrestrial Pole. This is a given Constant.

[dec]: The declination for these equations is expressed in terms of radians with respect to the Celestial Equator. The range is from [π/2] representing the Celestial North Pole to [-π/2] representing the South Celestial Pole. This is a given constant.

[obs]: The minimum altitude angle of an obstacle rising above a level plain that blocks the light of the Sun. For a level unobstructed plain; [obs = 0]. This angle is expressed in terms of radians. This is a given constant.

[hra]: The hour angle is expressed in terms of positive radians from nadir (midnight) to the following nadir (midnight). The range is from [0] to [2π]. This is the independent variable.

[ene]: The accumulated energy of the Sun as received on a level plain is expressed as [kw-hr/m^2].

[a]: The calculated positive altitude of the Sun with respect to a point directly above the site. This angle is expressed as radians. It has a range of [0] to [π].

[b]: The calculated positive latitude of the site with respect to the North Terrestrial Pole. This angle is expressed as radians. It has a range of [0] to [π].

[c]: The calculated positive declination of the Sun with respect to the North Celestial Pole. This angle is expressed as radians. It has a range of [0] to [π].

[d]: The calculated positive minimum altitude of an obstacle rising above a level plain that blocks the light of the Sun. This angle is expressed as radians. It has a range of [0] to [π].

[α]: The adjusted time of day with respect to the zenith of the Sun. This angle is expressed as radians. It has a range of [-π] to [+π].

[p]: This is a convenient calculated constant representing the product of the cosine of [b] and the cosine of [c].

[q]: This is a convenient calculated constant representing the product of the sine of [b] and the sine of [c].

[αr]: The time of “sunrise” with respect to the zenith of the Sun and the altitude of a “sunrise” obstacle. This is a negative [-] value.

[αs]: The time of “sunset” with respect to the zenith of the Sun and the altitude of a “sunset” obstacle. This is a positive [+] value.

[enr]: Constant for “sunrise” for integral equation.

[ens]: Sum of accumulated diurnal solar energy.

The following illustration depicts the solar energy formulae including a recapitulation of the solar power formulae.

lat = Given Site Latitude. dec = Given Declination of Sun.
sol = Given Solar Constant. obs = Given Obstruction Altitude.

$$b = \frac{\pi}{2} - lat \qquad c = \frac{\pi}{2} - dec \qquad p = \cos(b)\cdot\cos(c) \qquad q = \sin(b)\cdot\sin(c)$$

$$d = \frac{\pi}{2} - obs \qquad \alpha = hra - \pi \qquad a = acos(p + q\cdot\cos(\alpha)) \qquad pow = sol\cdot\cos(a)$$

$$pow = sol\cdot(p + q\cdot\cos(\alpha)) \qquad a < d$$

$$pow = 0 \qquad a \ge d$$

$$\cos(\alpha s) = \frac{\cos(d) - p}{q} \begin{cases} \alpha s = \pi & \cos(\alpha s) \le -1 \\ \alpha s = acos(\cos(\alpha s)) & -1 < \cos(\alpha s) < 1 \\ \alpha s = 0 & \cos(\alpha s) \ge 1 \end{cases}$$

$$\alpha r = -1\cdot\alpha s$$

$$enr = \int sol\cdot(p + q\cdot\cos(\alpha r))\, d\alpha r = \frac{-12\cdot sol}{\pi}\cdot(p\cdot\alpha r + q\cdot\sin(\alpha r))$$

$$ene = \int sol\cdot(p + q\cdot\cos(\alpha))\, d\alpha = \frac{12\cdot sol}{\pi}\cdot(p\cdot\alpha + q\cdot\sin(\alpha)) - enr \qquad \alpha r < \alpha < \alpha s$$

$$ens = 2\cdot\int sol\cdot(p + q\cdot\cos(\alpha s))\, d\alpha s = \frac{24\cdot sol}{\pi}\cdot(p\cdot\alpha s + q\cdot\sin(\alpha s))$$

This illustration depicts and example using the preceding power and energy formulae. Both the unrestricted and the restricted forms are shown.

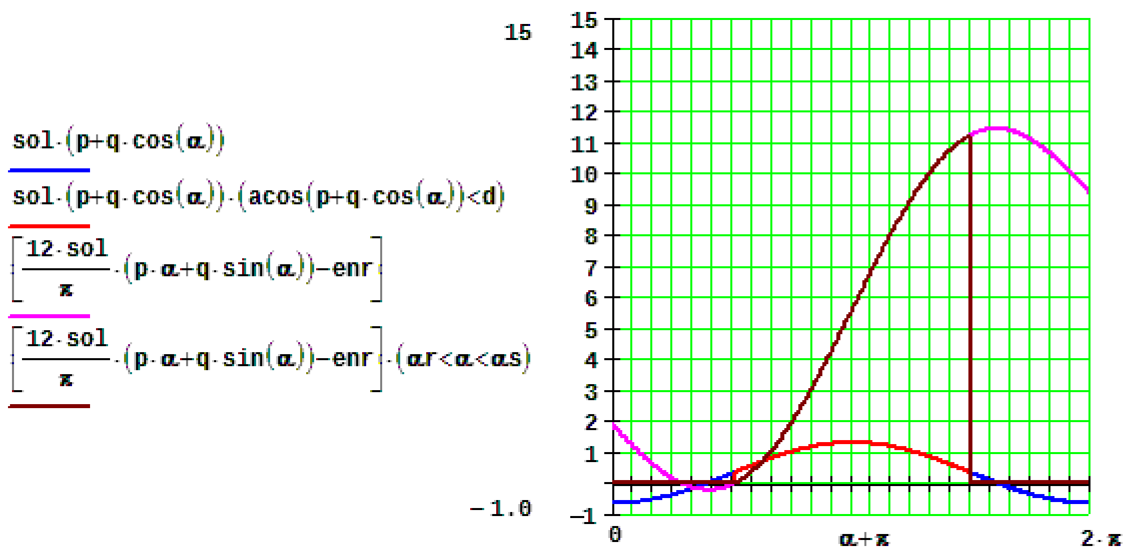
$$lat := 36.5 \cdot \frac{\pi}{180} \qquad dec := 23.45 \cdot \frac{\pi}{180} \qquad obs := 15 \cdot \frac{\pi}{180} \qquad sol := 1.33$$

$$b := \frac{\pi}{2} - lat \qquad c := \frac{\pi}{2} - dec \qquad d := \frac{\pi}{2} - obs$$

$$p := \cos(b) \cdot \cos(c) \qquad q := \sin(b) \cdot \sin(c)$$

$$\alpha s := \operatorname{acos}\left(\frac{\cos(d) - p}{q}\right) \qquad \alpha r := -1 \cdot \alpha s$$

$$enr := \frac{12 \cdot sol}{\pi} \cdot (p \cdot \alpha r + q \cdot \sin(\alpha r)) \qquad ens := \frac{24 \cdot sol}{\pi} \cdot (p \cdot \alpha s + q \cdot \sin(\alpha s)) = 11.195$$

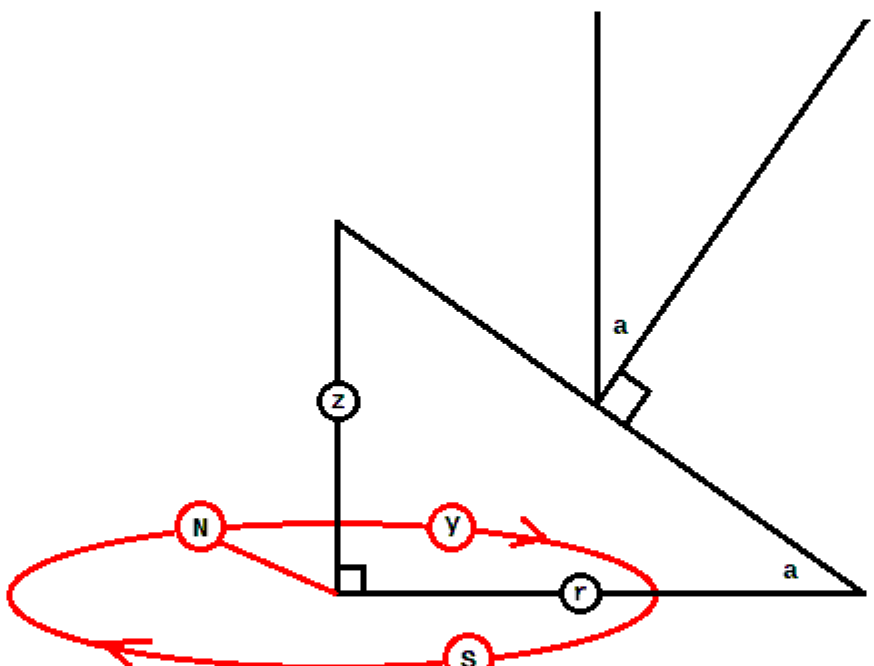


The brown trace represents the restricted form of the accumulated solar energy form. The magenta trace represents the unrestricted form. Observe the masking of the brown trace over the magenta trace.

An attempt has been made to write these formulae for any locale on the surface of the Earth, along with the requisite restrictions.

Chapter VII

Calculating the Displacement of Latitude and Longitude due to an Arbitrary Slope at the Locale of the Site



Chapter IV, Chapter V, and Chapter VI introduced the mathematical processes, equations, and formulae for determining the position, power, and energy of the Sun with respect to a level plain. This is a necessary baseline for solar calculations, but is not generally reflective of the real conditions for a specific site.

For most given sites on the Earth, there will be a well drained arbitrary slope. The well drained arbitrary slope will face onto an arbitrary compass azimuth. This situation is reflective of the real conditions for a specific site.

The arbitrary slope may be mathematically lifted from the surface of the Earth as a plane and transposed to another part of the surface of the Earth where it would be a level plain. Both the original sloped plane and the level transposed plane are parallel planes. This action results in a displacement of latitude and longitude. The mathematical displacement of the longitude is expressed as a displaced time differential.

A ray, perpendicular to the plane of the slope, may be projected upward onto the Celestial Sphere. From the point that the aforementioned ray intersects the Celestial Sphere, another ray perpendicular to the Celestial Sphere may be projected back down towards the center of the Earth. Where this second ray intersects the surface of the Earth, there will be a mathematically level plain that is perpendicular to the second ray. This mathematically displaced level plain will exist as a displacement of latitude and a displacement of longitude. The displacement of latitude will be with respect to the Terrestrial North Pole. The displacement of longitude will be with respect to the local prime meridian of the original site.

This is a fundamental principle used by navigators; both of land and of sea.

Calculating the Displacement of Latitude

Declaration of the Variables:

[lat]: The Terrestrial latitude of the Site with respect to the Terrestrial Equator. North latitude is assigned a positive [+] value. South latitude is assigned a negative [-] value.

[lad]: The displaced latitude with respect to the Terrestrial Equator. North displaced latitude is assigned a positive [+] value. South displaced latitude is assigned a negative [-] value.

[lod]: The displaced longitude with respect to the prime meridian of the site. East displaced longitude is assigned a positive [+] value. West displaced longitude is assigned a negative [-] value.

[r]: The positive horizontal run of the slope in any unit.

[z]: The positive vertical rise of the slope in the same unit as the run of the slope.

[y]: The positive compass azimuth of the facing of the slope.

[a]: The angle between a point directly above the site and a ray perpendicular to the slope.

[b]: The Terrestrial latitude of the site with respect to the Terrestrial North Pole.

[c]: The displaced latitude with respect to the Terrestrial North Pole.

[α]: The positive displaced longitude with respect to the prime meridian of the site.

lat = given r = given z = given γ = given

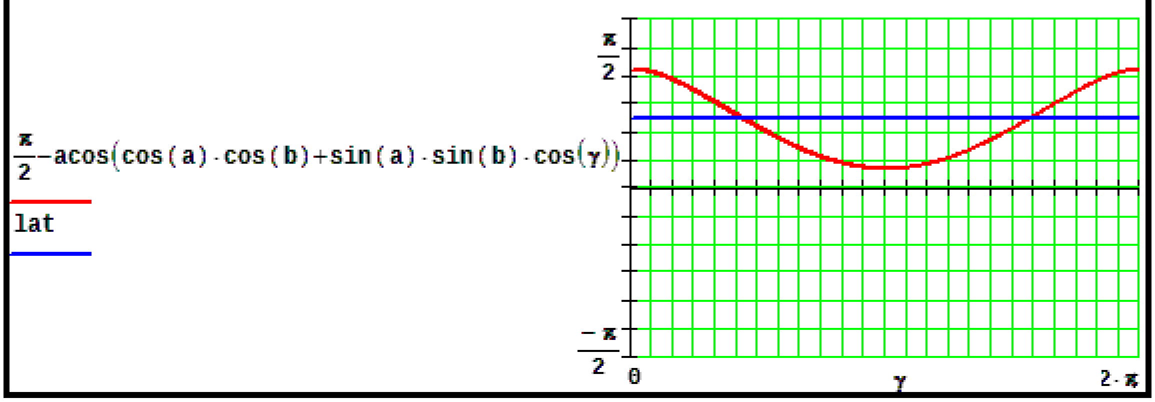
$$a = \operatorname{atan}\left(\frac{z}{r}\right) \quad r \neq 0 \quad \left\{ \begin{matrix} \text{Calculated} \\ \text{Constant} \end{matrix} \right\} \quad a = \frac{\pi}{2} \quad r = 0$$

$$b = \frac{\pi}{2} - lat \quad \left\{ \begin{matrix} \text{Calculated} \\ \text{Constant} \end{matrix} \right.$$

$$c = \operatorname{acos}(\cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma))$$

$$lad = \frac{\pi}{2} - c = \frac{\pi}{2} - \operatorname{acos}(\cos(a)\cdot\cos(b) + \sin(a)\cdot\sin(b)\cdot\cos(\gamma))$$

$$lat := 36.5\cdot\frac{\pi}{180} \quad r := 12 \quad z := 6 \quad a := \operatorname{atan}\left(\frac{z}{r}\right) \quad b := \frac{\pi}{2} - lat$$



Observe that the displaced latitude **MUST** be solved **BEFORE** the displaced longitude.

Calculating the Displacement of Longitude

The declaration of the variables are the same as given in "Calculating the Displacement of Longitude."

In this equation; angle [α] is solved as a positive [+] value. However, [lod] must be positive [+] when azimuth angle [γ] is to the East and must be negative [-] when azimuth angle [γ] is to the West.

lat = given   r = given   z = given   γ = given

$$a = \operatorname{atan}\left(\frac{z}{r}\right) \quad r \neq 0 \quad \left\{ \begin{array}{c}\text{Calculated}\\ \text{Constant}\end{array} \right\} \quad a = \frac{\pi}{2} \quad r = 0$$

$$b = \frac{\pi}{2} - lat \quad \left\{ \begin{array}{c}\text{Calculated}\\ \text{Constant}\end{array} \right.$$

$$c = \operatorname{acos}(\cos(a) \cdot \cos(b) + \sin(a) \cdot \sin(b) \cdot \cos(\gamma))$$

$$\alpha = \operatorname{acos}\left(\frac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}\right) \quad \left\{ \begin{array}{ll} lod = \alpha & \gamma < \pi \\ lod = -\alpha & \gamma \geq \pi \end{array} \right.$$

Observe that the solve angle [c] is inserted into the placeholder for [c] in the formula for [α]. Unless [c] has been previously solved angle [α] cannot be solved.

Angle [lod] represents the time differential between similar events on the level plain and on the arbitrary slope. For example; A time differential of [+2/π] will indicate that the Sun will reach zenith on the level plain 2 hours after it reaches zenith on the slope. Conversely; A time differential of [-2/π] will indicate that the Sun will reach zenith on the level plain 2 hours before it reaches zenith on the slope.

Here is an example of the preceding arguments as a function of the azimuth angle [γ]. The red trace represents the displaced latitude. The blue trace represents the site latitude. The solid magenta trace represents the easterly deviation of longitude for the displaced plane. The dashed magenta trace represents the westerly deviation of longitude for the displaced plane.

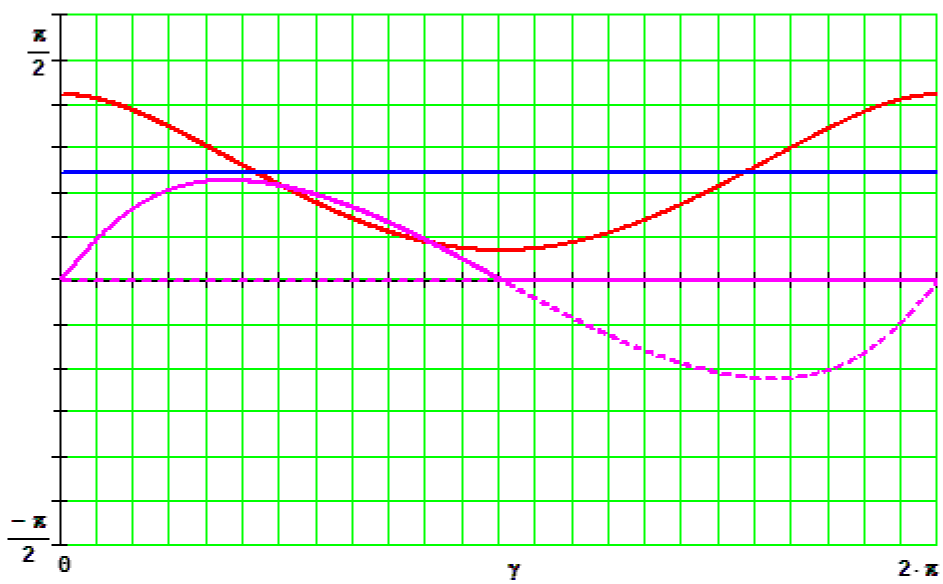
$$lat := 36.5 \cdot \frac{\pi}{180} \qquad r := 12 \qquad z := 6 \qquad a := atan\left(\frac{z}{r}\right) \qquad b := \frac{\pi}{2} - lat$$

$$\frac{\pi}{2} - acos(cos(a) \cdot cos(b) + sin(a) \cdot sin(b) \cdot cos(\gamma))$$

$$lat$$

$$acos\left(\frac{cos(a) - cos(b) \cdot cos(acos(cos(a) \cdot cos(b) + sin(a) \cdot sin(b) \cdot cos(\gamma)))}{sin(b) \cdot sin(acos(cos(a) \cdot cos(b) + sin(a) \cdot sin(b) \cdot cos(\gamma)))}\right)$$

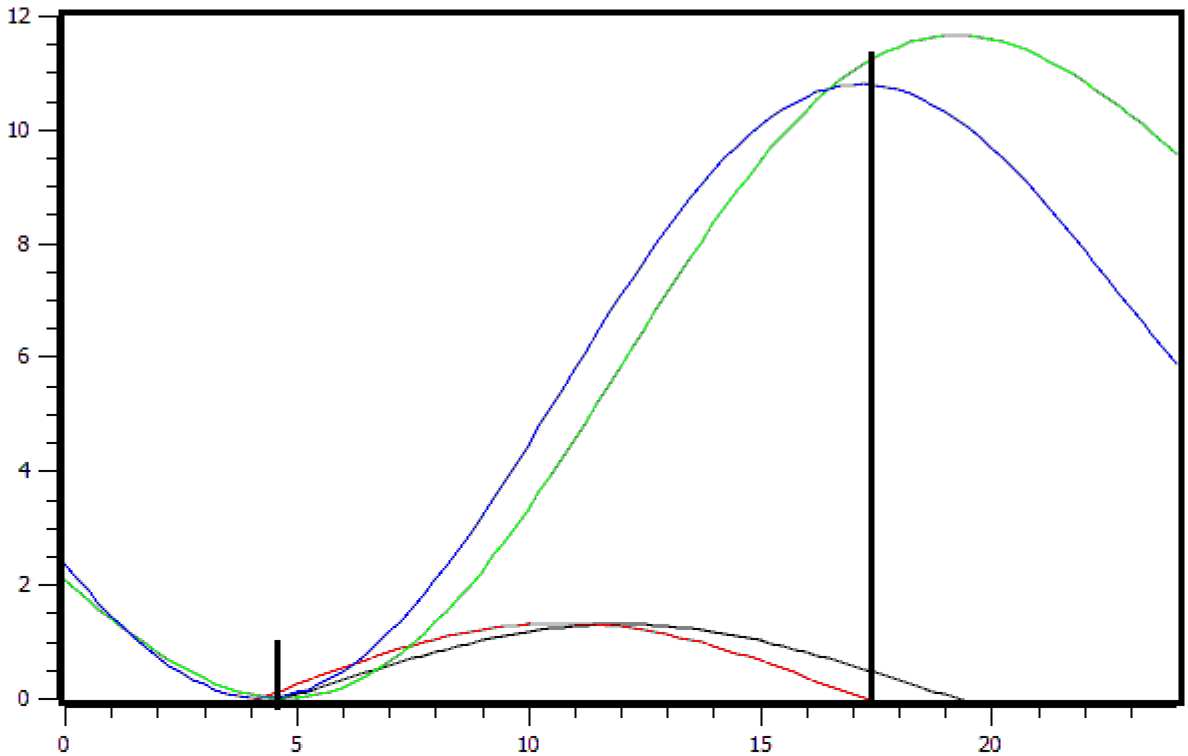
$$-acos\left(\frac{cos(a) - cos(b) \cdot cos(acos(cos(a) \cdot cos(b) + sin(a) \cdot sin(b) \cdot cos(\gamma)))}{sin(b) \cdot sin(acos(cos(a) \cdot cos(b) + sin(a) \cdot sin(b) \cdot cos(\gamma)))}\right)$$



Bear in mind that in order to acquire solar energy, the Sun must be simultaneously present on both the level plain and on the slope.

Chapter VIII

Calculating Solar Power and Solar Energy for the Arbitrary Slope



The basic calculations for solar power and solar energy for a level plain are given in Chapter VI. The calculations for the displacement of the latitude and the longitude are given in chapter VII. This chapter is concerned with the calculations for the solar power and solar energy acquired on an arbitrary slope at the locale of the site. This latter is represented by the displacement of the latitude and the longitude from chapter VII.

There are two set of calculations required. One is for the level plain at the site and the other is for an imaginary level plain located at the coordinates of the displaced latitude and longitude.

Due to a time difference, the two are in conflict. In order to acquire solar power and solar energy, the Sun must simultaneously shine on both the level plain and on the arbitrary slope. However, the level plain calculations for the arbitrary slope based on the displaced latitude have a time offset represented by the displaced longitude.

These calculations are a three part procedure. The first set of calculations is for the level plain of the site including an allowance for the obstruction altitude. The second set of calculations is for the imaginary unobstructed level plain at the locale of the displaced latitude. The third set of calculations is for the area of intersection of the first two sets of calculations, allowing for the time differential.

Let us now put the calculations of chapter VI and Chapter VII together. Let us now make the necessary choices. Much of the following will be a recapitulation of the chapter VI and Chapter VII.

This Chapter will essentially put everything on the table. The steps will be ordered like building a house from the ground up.

[01]: [latitude]

[latitude] as the first of the seven given parameters represents the Terrestrial latitude of the site with respect to the Terrestrial Equator. North [latitude] is assigned a positive [+] value. South [latitude] is assigned a negative [-] value. By convention [latitude] is expressed in terms of degrees.

[02]: [declination]

[declination] as the second of the seven given parameters represents the Celestial declination of the Sun with respect to the Celestial Equator. North [declination] is assigned a positive [+] value. South [declination] is assigned a negative [-] value. By convention [declination] is expressed in terms of degrees.

[03]: [obstruction]

[obstruction] as the third of the seven given parameters represents the minimum practical altitude of the Sun with respect to the level plain of the site. This is the minimum altitude required for the Sun to shine down on the site. The obstacle may be represented by trees, buildings, hillsides, mountains, or a politically shared common space. By convention [obstruction] is expressed in terms of degrees.420

[04]: [sol]

[sol] as the fourth of the seven given parameters represents the Solar Constant of the Sun for the given day. By convention [sol] is expressed in terms of kilowatts per square meter. Due to the elliptical orbit of the Earth, this “constant” varies slightly from 1.328 kilowatts per square meter to 1.420 kilowatts per square meter.

[05]: [rx]

[rx] as the fifth of the seven given parameters represents the positive [+] horizontal run of an arbitrary slope at the locale of the site. This may be expressed in any chosen unit of length. The [r] component of the name of the variable represents the radius of a circle where $r^2 = x^2 + y^2$. The [x] component of the name links it to the mathematics for the slope.

[06]: [zx]

[zx] as the sixth of the seven given parameters represents the positive [+] vertical rise of an arbitrary slope at the locale of the site. It must be expressed in the same units of length as [rx]. The [z] component of the name of the variable represents the [z] component in geometry that is used to express a height vector. The [x] component of the name links it to the mathematics for the slope.

[07]: [azimuthx]

[azimuthx] as the seventh and last of the seven given parameters represents the compass azimuth that the arbitrary slope at the locale of the site is facing. By convention it is expressed in terms of positive [+] clockwise degrees with respect to the direction of the Terrestrial North Pole. The [azimuth] component of the name represents the conventional understanding of “azimuth.” The [x] component of the name links it to the mathematics for the slope.

These are the seven given parameters required for calculating for solar power and solar energy. They are all in raw form. For doing the calculations; the angles all need to be expressed as radians.

[08]: [lat]

[lat] represents the Terrestrial latitude of the site as expressed in terms of radians. The trigonometric equations are all processed as radians. The conversion is from [latitude] in degrees to [lat] in radians. 1 radian is equal to (180/π) degrees.

$$lat = latitude \cdot \left(\frac{\pi}{180} \right)$$

[09]: [dec]

[dec] represents the Celestial declination of the Sun as expressed in terms of radians. The trigonometric equations are all processed as radians. The conversion is from [declination] in degrees to [dec] in radians. 1 radian is equal to (180/π) degrees.

$$dec = declination \cdot \left(\frac{\pi}{180} \right)$$

[10]: [obs]

[obs] represents the obstruction altitude at the site as expressed in terms of radians. The trigonometric equations are all processed as radians. The conversion is from [obstruction] in degrees to [obs] in radians. 1 radian is equal to (180/π) degrees.

$$obs = obstruction \cdot \left(\frac{\pi}{180} \right)$$

[11]: [azix]

[azix] represents the azimuth at the slope as expressed in terms of radians. The trigonometric equations are all processed as radians. The conversion is from [azimuthx] in degrees to [azix] in radians. 1 radian is equal to (180/π) degrees. The [x] at the end of the name of the variable links it to the slope.

$$azix = azimuthx \cdot \left(\frac{\pi}{180}\right)$$

[12]: [b]

[b] represents the positive latitude of the site with respect to the Terrestrial North Pole. This variable is expressed here in terms of radians for mathematical processing. [b] has a range of 0 to π.

$$b = \frac{\pi}{2} - lat$$

[13]: [c]

[c] represents the positive declination of the Sun with respect to the Celestial North Pole. This variable is expressed here in terms of radians for mathematical processing. [c] has a range of 0 to π.

$$c = \frac{\pi}{2} - dec$$

[14]: [d]

[d] represents the positive obstruction altitude with respect to a point directly above the site. This variable is expressed here in terms of radians for mathematical processing. [d] has a practical range of 0 to π/2. This variable is akin to the applications of [a] as per spherical trigonometry.

$$d = \frac{\pi}{2} - obs$$

[15]: [ax]

[ax] represents the altitude angle of the slope with respect to the level horizon of the site, or alternately, the angle of a ray perpendicular to the slope with respect to a ray originating at the center of the Earth and passing through the site.

$$ax = \operatorname{atan}\left(\frac{zx}{rx}\right) \qquad rx \neq 0$$

$$ax = \frac{\pi}{2} \qquad rx = 0$$

[16]: [bx]

[bx] represents the mathematically imaginary displaced latitude of the slope with respect to the Terrestrial North Pole. The power and energy calculations for the slope require this variable.

$$bx = \operatorname{acos}(\cos(b) \cdot \cos(ax) + \sin(b) \cdot \sin(ax) \cdot \cos(azix))$$

[17]: [αx]

[αx] represents the deviation of the imaginary displacement of longitude with respect to the meridian passing through the locale of the site. In the conventional nomenclature; [αx] is assigned a positive [+] value when it is to the East and a negative [-] value when it is assigned to the West. For the sake of the calculations, it is expressed in terms of radians. The vital importance of [αx] is that it represents a time differential between the level obstructed plain of the site and the displaced plane of the slope.

$$\alpha x = \operatorname{acos}\left(\frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}\right) \qquad azix \le \pi$$

$$\alpha x = -\operatorname{acos}\left(\frac{\cos(ax) - \cos(b)\cdot\cos(bx)}{\sin(b)\cdot\sin(bx)}\right) \qquad azix > \pi$$

This completes the ordered list of the seven original given parameters as well as the subsequent calculated operational parameters. The latter is given in a the logical order similar to building a house from the ground up.

The next two items are more of a curiosity than needful.

[18]: [latitudex]

[latitudex] represents the displaced latitude due to the slope with respect to the Terrestrial Equator as expressed in terms of degrees.

$$latitudex = \left(\frac{\pi}{2} - bx\right)\cdot\frac{180}{\pi}$$

[19]: [longitudex]

[longitudex] represents the displaced longitude due to the slope with respect to the meridian passing through the site as expressed in terms of degrees.

$$longitudex = \alpha x \cdot \frac{180}{\pi}$$

The next four items are used as convenient constants in order to reduce clutter and clarify the calculations.

[20]: [p]

[p] represents a compacted constant comprising the product of the cosine of [b] and the cosine of [c]. [p] is paired with [q] in calculations pertaining to the level obstructed plain of the site.

$$p = \cos(b) \cdot \cos(c)$$

[21]: [q]

[q] represents a compacted constant comprising the product of the sine of [b] and the sine of [c]. [q] is paired with [p] in calculations pertaining to the level obstructed plain of the site.

$$q = \sin(b) \cdot \sin(c)$$

[22]: [px]

[px] represents a compacted constant comprising the product of the cosine of [bx] and the cosine of [c]. [px] is paired with [qx] in calculations pertaining to the arbitrary slope.

$$px = \cos(bx) \cdot \cos(c)$$

[23]: [qx]

[qx] represents a compacted constant comprising the product of the sine of [bx] and the sine of [c]. [qx] is paired with [px] in calculations pertaining to the arbitrary slope.

$$qx = \sin(bx) \cdot \sin(c)$$

It is essential to know the times of sunrise and sunset for both the level obstructed plain of the site and the displaced plane of the slope. The following six calculations are employed to determine the times of sunrise and sunset with respect to the times of zenith of both the level obstructed plain and the displaced plane of the slope. No allowance is made for the displaced longitude of the slope. That will come later.

These calculations are important because there can be no solar power or solar energy on either plane if the Sun is below the obstacle altitude. In addition, there can be no solar power or solar energy if the Sun is not simultaneously shining down on both planes.

[24]: [αt]

[αt] represents a test for the time of sunrise and sunset for the level obstructed plain of the site with respect to the time of zenith for the level obstructed plain of the site. In a nutshell, it is the cosine of the applicable time of sunrise for the level obstructed plain of the site. The [α] component pertains to the rotation of the Sun about the Celestial North Pole and the [t] component indicates a test.

$$\alpha t = \frac{\cos(d) - p}{q}$$

[25}: [αr]

[αr] represents the applicable time of sunrise for the level obstructed plain of the site with respect to the time of zenith for the level obstructed plain of the site. This is always a negative [-] value. There must be a time of sunrise. However, if the sun is always above the obstacle altitude, then [αt < -1] and the time of sunrise is assigned a value of [-π]. If the Sun is always below the obstacle altitude, then [αt > +1] and the time of sunrise is assigned a value of [0]. If [αt] is within the range permitted for the arc-cosine, then [αr] is equal to the negative [-] arc-cosine of [αt].

$$\alpha r = -\pi \qquad \alpha t \le -1$$

$$\alpha r = -\mathrm{acos}(\alpha t) \qquad -1 < \alpha t < 1$$

$$\alpha r = 0 \qquad \alpha t \ge 1$$

[26}: [αs]

[αs] represents the applicable time of sunset for the level obstructed plain of the site with respect to the time of zenith for the level obstructed plain of the site. This is always a positive [+] value. There must be a time of sunset. However, if the sun is always above the obstacle altitude, then [αt < -1] and the time of sunset is assigned a value of [+π]. If the Sun is always below the obstacle altitude, then [αt > +1] and the time of sunset is assigned a value of [0]. If [αt] is within the range permitted for the arc-cosine, then [αs] is equal to the positive [+] arc-cosine of [αt].

$$\alpha s = \pi \qquad \alpha t \le -1$$

$$\alpha s = \operatorname{acos}(\alpha t) \qquad -1 < \alpha t < 1$$

$$\alpha s = 0 \qquad \alpha t \ge 1$$

[27]: [αxt]

[αxt] represents a test for the time of sunrise and sunset for the level displaced plane of the slope with respect to the time of zenith for the level displaced plane of the slope. The [α] component pertains to the rotation of the Sun about the Celestial North Pole, The [x] component pertains to the displaced plane. The [t] component indicates a test. [αxt] assumes that the Sun will rise and set at the level unobstructed horizon of the displaced plane of the slope from the viewpoint of the displaced plane of the slope.

$$\alpha xt = \frac{0 - px}{qx}$$

[28}: [αxr]

[αxr] represents the applicable time of sunrise for the level unobstructed plane of the slope with respect to the time of zenith for the level unobstructed plane of the slope. This is always a negative [-] value. There must be a time of sunrise. However, if the sun is always above the level horizon of the plane of the slope, then [αxt < -1] and the time of sunrise is assigned a value of [-π]. If the sun is always below the level horizon of the plane of the slope, then [αxt > +1] and the time of sunrise is assigned a value of [0]. If [αxt] is within the range permitted for the arc-cosine, then [αxr] is equal to the negative [-] arc-cosine of [αxt].

$$\alpha xr = -\pi \qquad \alpha xt \le -1$$

$$\alpha xr = -\operatorname{acos}(\alpha xt) \qquad -1 < \alpha xt < 1$$

$$\alpha xr = 0 \qquad \alpha xt \ge 1$$

[29}: [αxs]

[αxs] represents the applicable time of sunset for the level unobstructed plane of the slope with respect to the time of zenith for the level unobstructed plane of the slope. This is always a positive [+] value. This is a positive variant of [αxr]

$$\alpha xs = \pi \qquad \alpha xt \le -1$$

$$\alpha xs = \operatorname{acos}(\alpha xt) \qquad -1 < \alpha xt < 1$$

$$\alpha xs = 0 \qquad \alpha xt \ge 1$$

These next four calculations are more of academic interest than functional in determining the power and energy potentials for the arbitrary slope. The returns are all with respect to the conventional 24 hour clock beginning at midnight on the level obstructed plain of the site.

The times of sunrise and sunset for the level obstructed plain are for the times that the Sun is at the obstruction altitude. The times of sunrise and sunset for the slope are for the times that the Sun is at the level imaginary horizon of the plane of the slope.

The applicable times of sunrise or sunset for the slope is whichever of the pairs in closest to noon [12 hr].

[30]: [sunrise]

[sunrise] represents the time of sunrise for the obstructed level plain of the site as expressed in terms of the conventional 24 hour clock beginning at midnight.

$$\mathbf{sunrise} = (\pi + \alpha r) \cdot \frac{12}{\pi}$$

[31]: [sunset]

[sunset] represents the time of sunset for the obstructed level plain of the site as expressed in terms of the conventional 24 hour clock beginning at midnight.

$$\mathbf{sunset} = (\pi + \alpha s) \cdot \frac{12}{\pi}$$

[32]: [sunrisex]

[sunrisex] represents the time of sunrise for the slope with respect to the meridian passing through the site as expressed in terms of the conventional 24 hour clock beginning at midnight.

$$\text{sunrisex} = \left[\pi + (\alpha xr - \alpha x)\right] \cdot \frac{12}{\pi}$$

[33]: [sunsetx]

[sunsetx] represents the time of sunset for the slope with respect to the meridian passing through the site as expressed in terms of the conventional 24 hour clock beginning at midnight.

$$\text{sunsetx} = \left[\pi + (\alpha xs - \alpha x)\right] \cdot \frac{12}{\pi}$$

In order to predict the potential solar power and accumulated solar energy for the slope, a time of sunrise and a time of sunset must be selected. This selection should be mathematically defined. This selection should be from the perspective of the displacement of the latitude and the displacement of the longitude due to the arbitrary slope.

These next for calculations are selecting the applicable time of sunrise and sunset for the Sun as it shines down on the plane of the arbitrary slope. This is done from the perspective of the displaced latitude and the displaced longitude.

[34]: [αxrs]

[αxrs] represents the applicable time of “sunrise” for the arbitrary slope. The perspective is from the arbitrary slope. The letter [s] at the end denotes “selected.” There are two options. The option that is nearest to the apparent zenith of the Sun is the applicable choice.

$$\alpha xrs = \alpha r + \alpha x \qquad (\alpha r + \alpha x) \geq \alpha xr$$

$$\alpha xrs = \alpha xr \qquad (\alpha r + \alpha x) < \alpha xr$$

[35]: [αxss]

[αxss] represents the applicable time of “sunset” for the arbitrary slope. The perspective is from the arbitrary slope. The letter [s] at the end denotes “selected.” There are two options. The option that is nearest to the apparent zenith of the Sun is the applicable choice.

$$\alpha xss = \alpha s + \alpha x \qquad (\alpha s + \alpha x) < \alpha xs$$

$$\alpha xss = \alpha xs \qquad (\alpha s + \alpha x) \geq \alpha xs$$

These two selections are for the times of “sunrise” and “sunset” in the arbitrary slope. They both are critical for calculating the constants to the accumulated solar energy integral for the arbitrary slope.

For the obstructed level plain of the site, the constants for the accumulated solar energy integral are more straight-forward. The times of “sunrise” and “sunset” are for the times that the altitude of the Sun is at the obstruction altitude. That is to say; [αr] and [αs] without the need for a selection.

These next six entries are for the diurnal accumulated solar energy both of the obstructed level plain of the site and of the slope. The names of all of these six variables are prefixed with “en” pertaining to “energy.” The units for the accumulated solar energy are expressed as kilowatt-hours per square meter.

[36]: [enr]

[enr] represents the negative [-] value that the energy integral for the level obstructed plain of the site will yield for the time that the altitude of the rising Sun is at the same altitude as the obstacle altitude. When this negative [-] value is subtracted from the negative [-] value of the same, the result is zero. Thus, [enr] acts as the requisite constant to the main accumulated energy integral.

$$enr = \frac{12}{\pi} \cdot sol \cdot \left(\alpha r \cdot p + q \cdot \sin(\alpha r)\right)$$

[37]: [ens]

[ens] represents the positive [+] value that the energy integral for the level obstructed plain of the site will yield for the time that the altitude of the setting Sun is at the same altitude as the obstacle altitude. Since [obs] applies equally to both the rising and setting of the Sun, [ens] will be a positive [+] version of the negative [-] [enr].

$$ens = \frac{12}{\pi} \cdot sol \cdot \left(\alpha s \cdot p + q \cdot \sin(\alpha s)\right)$$

[38]: [ent]

[ent] represents the total accumulated solar energy for the period of the day from midnight to the midnight for the obstructed level plain of the site. Technically speaking, this could be simply written as **[ent = ens – enr]**; but because **[ens = -enr]**, it may be written as **[ent = 2 · ens]**.

$$ent = \frac{24}{\pi} \cdot sol \cdot (\alpha s \cdot p + q \cdot \sin(\alpha s))$$

[39]: [enxr]

[enxr] represents the value of the energy integral for the level unobstructed plane of the slope at the selected time of the rising of the Sun. This is an essential constant for the general energy integral.

$$enxr = \frac{12}{\pi} \cdot sol \cdot (\alpha xrs \cdot px + qx \cdot \sin(\alpha xrs))$$

[40]: [enxs]

[enxs] represents the value of the energy integral for the level unobstructed plane of the slope at the selected time of the setting of the Sun.

$$enxs = \frac{12}{\pi} \cdot sol \cdot (\alpha xss \cdot px + qx \cdot \sin(\alpha xss))$$

[41]: [enxt]

[enxt] represents the total maximum possible accumulated solar energy for the period of the day from midnight to the midnight for the unobstructed plain of the slope nested in the obstructed plane of the site.

$$enxt = enxs - enxr$$

This completes the list of sequential calculations for the summary of the solar energy equations. Both the calculations for the level plain of the obstructed site and for the arbitrary slope. The proper allowance has been given to eliminate the extraneous results.

What follows next are the calculations for the tabular data for power and energy as a function of the time of the day with respect to the nadir of the Sun. The result for both the level plain of the site and for the arbitrary slope are given; the former for reference and the latter within the margins of the former.

If the available solar power is less than or equal to zero, then it will be assigned a value of zero. The applicable time margins will be the guide.

If the available accumulated solar energy is out-of-bounds, it will be assigned a value of [0] for the morning and a value of [ent] or [enxt] respectfully] Before the applicable time of the respective risings of the Sun, no solar energy from midnight has been garnered. After the applicable time of the respective settings of the Sun and on to midnight, all of the energy of the Sun that could be accumulated for the day has been accumulated for the day; in which case it is assigned a value equal to the total accumulated value.

[42]: [hour]

[hour] represents the given conventional time of the solar day with respect to the solar midnight. The units are in hours as suggested by the name of the variable. This is a positive value with a range of 0 hours to 24 hours. 0 and 24 hours represents the nadir of the Sun. 12 hours represents the zenith of the Sun.

$$hour = given \qquad 0 \le hour < 24$$

[43]: [hra]

[hra] represents the positive [+] hour angle of the Sun from nadir. This is simply a conversion of hours to radians. The range of [hra] is from 0 to 2π.

$$hra = hour \cdot \frac{\pi}{12} \qquad 0 \le hra < 2 \cdot \pi$$

[44]: [α]

[α] represents the time of the solar day with respect to the zenith of the Sun. The units are in radians. The range is from [-π] to [+π]. The morning is assigned a negative [-] value and the afternoon is assigned a positive [+] value. [α] is the essential independent variable for the tabular calculations.

$$\alpha = hra - \pi \qquad -\pi \le \alpha < \pi$$

Here we have the sequential calculations for the altitude of the Sun throughout the day. The results from [a] will be fed back into the azimuth calculation.

[45]: [a]

[a] represents the altitude of the Sun with respect to a point directly above the site. [a] is expressed in terms of radians. This is always a positive [+] value with a range of [0] to [+π].

$$a = \operatorname{acos}(p + q \cdot \cos(\alpha))$$

[46]: [alt]

[alt] represents the altitude of the Sun with respect to the level unobstructed horizon of the level plain of the site. [alt] is expressed in terms of radians with a range of [-π] to [+π].

$$alt = \frac{\pi}{2} - a$$

[47]: [altitude]

[altitude] represents the altitude of the Sun with respect to the level unobstructed horizon of the level plain of the site as well. [altitude] is expressed in terms of conventional degrees. It has a range of [-90°] to [+90°].

$$altitude = alt \cdot \frac{180}{\pi}$$

These next three calculations are for the azimuth angle of the Sun. In a tabular report for the power and energy of the Sun; the altitude angle and the azimuth angle are both of interest.

[48]: [γ]

[γ] represents the azimuth angle of the Sun with respect to the direction of the Terrestrial North Pole. [γ] is expressed in terms of radians with a range of [-π] to [+π]. East is assigned a positive [=] value and West is assigned a negative [-] value.

$$\gamma = \operatorname{acos}\left(\frac{\cos(c) - \cos(a)\cdot\cos(b)}{\sin(a)\cdot\sin(b)}\right)$$

[49]: [azi]

[azi] represents the azimuth angle of the Sun with respect to the direction of the Terrestrial North pole. [azi] is expressed in terms of radians with a range of [0] to [2π]. This is a positive adjustment of [γ].

$$\begin{aligned} azi &= \gamma & \alpha \le \pi \\ azi &= 2\cdot\pi - \gamma & \alpha > \pi \end{aligned}$$

[50]: [azimuth]

[azimuth] represents the azimuth of the Sun with respect to the direction of the Terrestrial North pole. [azimuth] is expressed in terms of conventional degrees with a range of [0°] to [360°].

$$azimuth = azi \cdot \frac{180}{\pi}$$

Here are the calculations for the solar power potentials for both the level obstructed plain of the site and the nested arbitrary slope.

[51]: [pw]

[pw] represents the power, i.e. the intensity, of the energy received on the level obstructed plain of the site. It is expressed as kilowatts per square meter. When the light of the Sun is blocked by an obstacle or by the main body of the Earth, i.e. nighttime, there is no solar power available. When no solar power is available, the solar power is assigned a value of zero. Negative [-] power is not possible.

$$\begin{aligned} pw &= sol \cdot (p + q \cdot \cos(\alpha)) && \alpha r < \alpha < \alpha s \\ pw &= 0 && (\alpha \le \alpha r) \vee (\alpha \ge \alpha s) \end{aligned}$$

[52]: [pwx]

[pwx] represents the power, i.e. the intensity, of the energy received on the arbitrarily sloped plane of the arbitrary slope at the locale of the site. It is expressed as kilowatts per square meter. [pwx] represents from the perspective of the slope; the intersection of the available power for the level obstructed plain of the site and the available power for the plane of the arbitrary plane. It is defined by the applicable time of sunrise and the applicable time of sunset.

$$\begin{aligned} pwx &= sol \cdot (px + qx \cdot \cos(\alpha + \alpha x)) && (\alpha xrs - \alpha x) < \alpha < (\alpha xss - \alpha x) \\ pwx &= 0 && \alpha \le (\alpha xrs - \alpha x) \vee \alpha \ge (\alpha xss - \alpha x) \end{aligned}$$

[53]: [en]

[en] represents the accumulated available solar energy for the level obstructed plain of the site. This is expressed as kilowatt-hours per square meter. If the Sun has not yet risen on the site, then [en] is assigned a value of zero. If the Sun has set on the site, then [en] is assigned a value equal to the total accumulated solar energy for the day.

$$en = 0 \qquad \alpha \le \alpha r$$

$$en = \frac{12}{\pi} \cdot sol \cdot (\alpha \cdot p + q \cdot \sin(\alpha)) - enr \qquad \alpha r < \alpha < \alpha s$$

$$en = ent \qquad \alpha \ge \alpha s$$

[54]: [enx]

[enx] represents the accumulated available solar energy for the arbitrary slope. This is expressed as kilowatt-hours per square meter. If the Sun has not yet risen on the slope, then [enx] is assigned a value of zero. If the Sun has set on the slope, then [enx] is assigned a value equal to the total accumulated solar energy for the day.

$$enx = 0 \qquad \alpha \le (\alpha xrs - \alpha x)$$

$$(\alpha xrs - \alpha x) < \alpha < (\alpha xss - \alpha x)$$

$$enx = \frac{12}{\pi} \cdot sol \cdot [(\alpha + \alpha x) \cdot px + qx \cdot \sin(\alpha + \alpha x)] - enxr$$

$$enx = enxt \qquad \alpha \ge (\alpha xss - \alpha x)$$

At this point in time your mind is befuddled, confused, and bemused. Do not despair. This chapter is the conclusion of the direct solar power and solar energy calculations. In presenting the series of calculations for the solar power and solar energy for the arbitrary slope, it was necessary to present the foundations from the original given parameters.

Let us now look at some examples of these calculations in a tabular format. This will include a summary and a table at 1 hour increments beginning at midnight and ending at the following midnight.

There will be two different latitudes. Each latitude will have have the same obstruction altitude and the same parameters for the arbitrarily slope. There will be three different times of the year represented.

```
Terrestrial Latitude of the site ------------ :   85.000 deg.
Celestial Declination of the Sun of the Sun - :   23.450 deg.
Obstruction Altitude ------------------------ :   15.000 deg.
Solar Constant ------------------------------ :    1.330 kw/m^2.
Horizontal Run of Slope --------------------- :   12.000 unit(s).
Vertical Rise of Slope ---------------------- :    6.000 unit(s).
Compass Azimuth of Slope -------------------- :  135.000 deg.
Displaced Latitude for Slope ---------------- :   59.708 deg.
Displaced Longitude for Slope --------------- :   38.824 deg.
Time of Sunrise for Level Plain of Site ----- :    0.000 hr.
Time of Sunset for Level Plain of Site ------ :   24.000 hr.
Time of Sunrise for Plane of Slope ---------- :    0.215 hr.
Time of Sunset for Plane of Slope ----------- :   18.608 hr.
Diurnal Solar Energy for Level Plain of Site  :   12.654 kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :   11.555 kw-hr/m^2.
```

Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	18.45	0.000	0.000	0.000	0.000	0.000
1.00	18.61	14.510	0.425	0.094	0.422	0.036
2.00	19.10	29.040	0.435	0.235	0.851	0.198
3.00	19.87	43.612	0.452	0.391	1.295	0.510
4.00	20.88	58.251	0.474	0.551	1.757	0.981
5.00	22.07	72.982	0.500	0.706	2.244	1.611
6.00	23.36	87.835	0.527	0.843	2.757	2.387
7.00	24.65	102.833	0.555	0.954	3.298	3.288
8.00	25.88	117.991	0.580	1.031	3.866	4.283
9.00	26.93	133.312	0.602	1.069	4.458	5.336
10.00	27.75	148.779	0.619	1.065	5.069	6.407
11.00	28.27	164.359	0.630	1.020	5.695	7.453
12.00	28.45	180.000	0.634	0.937	6.327	8.434
13.00	28.27	195.641	0.630	0.820	6.959	9.315
14.00	27.75	211.221	0.619	0.679	7.585	10.067
15.00	26.93	226.688	0.602	0.523	8.196	10.669
16.00	25.88	242.009	0.580	0.363	8.788	11.111
17.00	24.65	257.167	0.555	0.208	9.356	11.396
18.00	23.36	272.165	0.527	0.071	9.897	11.534
19.00	22.07	287.018	0.500	0.000	10.410	11.555
20.00	20.88	301.749	0.474	0.000	10.897	11.555
21.00	19.87	316.388	0.452	0.000	11.360	11.555
22.00	19.10	330.960	0.435	0.000	11.803	11.555
23.00	18.61	345.490	0.425	0.000	12.232	11.555
24.00	18.45	360.000	0.000	0.000	12.654	11.555

```
Terrestrial Latitude of the site ------------ :   85.000 deg.
Celestial Declination of the Sun of the Sun - :    0.000 deg.
Obstruction Altitude ------------------------ :   15.000 deg.
Solar Constant ------------------------------ :    1.360 kw/m^2.
Horizontal Run of Slope --------------------- :   12.000 unit(s).
Vertical Rise of Slope ---------------------- :    6.000 unit(s).
Compass Azimuth of Slope -------------------- :  135.000 deg.
Displaced Latitude for Slope ---------------- :   59.708 deg.
Displaced Longitude for Slope --------------- :   38.824 deg.
Time of Sunrise for Level Plain of Site ----- :   12.000 hr.
Time of Sunset for Level Plain of Site ------ :   12.000 hr.
Time of Sunrise for Plane of Slope ---------- :   12.000 hr.
Time of Sunset for Plane of Slope ----------- :   12.000 hr.
Diurnal Solar Energy for Level Plain of Site  :    0.000 kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :    0.000 kw-hr/m^2.
```

Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	-5.00	0.000	0.000	0.000	0.000	0.000
1.00	-4.83	15.055	0.000	0.000	0.000	0.000
2.00	-4.33	30.095	0.000	0.000	0.000	0.000
3.00	-3.53	45.109	0.000	0.000	0.000	0.000
4.00	-2.50	60.095	0.000	0.000	0.000	0.000
5.00	-1.29	75.055	0.000	0.000	0.000	0.000
6.00	0.00	90.000	0.000	0.000	0.000	0.000
7.00	1.29	104.946	0.000	0.000	0.000	0.000
8.00	2.50	119.906	0.000	0.000	0.000	0.000
9.00	3.53	134.891	0.000	0.000	0.000	0.000
10.00	4.33	149.905	0.000	0.000	0.000	0.000
11.00	4.83	164.945	0.000	0.000	0.000	0.000
12.00	5.00	180.000	0.000	0.000	0.000	0.000
13.00	4.83	195.055	0.000	0.000	0.000	0.000
14.00	4.33	210.095	0.000	0.000	0.000	0.000
15.00	3.53	225.109	0.000	0.000	0.000	0.000
16.00	2.50	240.094	0.000	0.000	0.000	0.000
17.00	1.29	255.054	0.000	0.000	0.000	0.000
18.00	0.00	270.000	0.000	0.000	0.000	0.000
19.00	-1.29	284.945	0.000	0.000	0.000	0.000
20.00	-2.50	299.905	0.000	0.000	0.000	0.000
21.00	-3.53	314.891	0.000	0.000	0.000	0.000
22.00	-4.33	329.905	0.000	0.000	0.000	0.000
23.00	-4.83	344.945	0.000	0.000	0.000	0.000
24.00	-5.00	360.000	0.000	0.000	0.000	0.000

```
Terrestrial Latitude of the site ------------ :   85.000 deg.
Celestial Declination of the Sun of the Sun - :  -23.450 deg.
Obstruction Altitude ------------------------ :   15.000 deg.
Solar Constant ------------------------------ :    1.420 kw/m^2.
Horizontal Run of Slope --------------------- :   12.000 unit(s).
Vertical Rise of Slope ---------------------- :    6.000 unit(s).
Compass Azimuth of Slope -------------------- :  135.000 deg.
Displaced Latitude for Slope ---------------- :   59.708 deg.
Displaced Longitude for Slope --------------- :   38.824 deg.
Time of Sunrise for Level Plain of Site ----- :   12.000 hr.
Time of Sunset for Level Plain of Site ------ :   12.000 hr.
Time of Sunrise for Plane of Slope ---------- :   12.000 hr.
Time of Sunset for Plane of Slope ----------- :   12.000 hr.
Diurnal Solar Energy for Level Plain of Site  :    0.000 kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :    0.000 kw-hr/m^2.
```

Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	-28.45	0.000	0.000	0.000	0.000	0.000
1.00	-28.27	15.641	0.000	0.000	0.000	0.000
2.00	-27.75	31.221	0.000	0.000	0.000	0.000
3.00	-26.93	46.689	0.000	0.000	0.000	0.000
4.00	-25.88	62.010	0.000	0.000	0.000	0.000
5.00	-24.65	77.168	0.000	0.000	0.000	0.000
6.00	-23.36	92.165	0.000	0.000	0.000	0.000
7.00	-22.07	107.018	0.000	0.000	0.000	0.000
8.00	-20.88	121.750	0.000	0.000	0.000	0.000
9.00	-19.87	136.388	0.000	0.000	0.000	0.000
10.00	-19.10	150.960	0.000	0.000	0.000	0.000
11.00	-18.61	165.490	0.000	0.000	0.000	0.000
12.00	-18.45	180.000	0.000	0.000	0.000	0.000
13.00	-18.61	194.510	0.000	0.000	0.000	0.000
14.00	-19.10	209.040	0.000	0.000	0.000	0.000
15.00	-19.87	223.612	0.000	0.000	0.000	0.000
16.00	-20.88	238.250	0.000	0.000	0.000	0.000
17.00	-22.07	252.982	0.000	0.000	0.000	0.000
18.00	-23.36	267.835	0.000	0.000	0.000	0.000
19.00	-24.65	282.832	0.000	0.000	0.000	0.000
20.00	-25.88	297.990	0.000	0.000	0.000	0.000
21.00	-26.93	313.311	0.000	0.000	0.000	0.000
22.00	-27.75	328.779	0.000	0.000	0.000	0.000
23.00	-28.27	344.359	0.000	0.000	0.000	0.000
24.00	-28.45	360.000	0.000	0.000	0.000	0.000

```
Terrestrial Latitude of the site ------------ :   36.500 deg.
Celestial Declination of the Sun of the Sun - :   23.450 deg.
Obstruction Altitude ------------------------ :   15.000 deg.
Solar Constant ------------------------------ :    1.330 kw/m^2.
Horizontal Run of Slope --------------------- :   12.000 unit(s).
Vertical Rise of Slope ---------------------- :    6.000 unit(s).
Compass Azimuth of Slope -------------------- :  135.000 deg.
Displaced Latitude for Slope ---------------- :   16.130 deg.
Displaced Longitude for Slope --------------- :   19.220 deg.
Time of Sunrise for Level Plain of Site ----- :    6.115 hr.
Time of Sunset for Level Plain of Site ------ :   17.885 hr.
Time of Sunrise for Plane of Slope ---------- :    6.115 hr.
Time of Sunset for Plane of Slope ----------- :   17.199 hr.
Diurnal Solar Energy for Level Plain of Site  :   11.195 kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :   10.253 kw-hr/m^2.
```

Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	-30.05	0.000	0.000	0.000	0.000	0.000
1.00	-28.40	15.660	0.000	0.000	0.000	0.000
2.00	-23.70	30.063	0.000	0.000	0.000	0.000
3.00	-16.54	42.588	0.000	0.000	0.000	0.000
4.00	-7.59	53.275	0.000	0.000	0.000	0.000
5.00	2.63	62.509	0.000	0.000	0.000	0.000
6.00	13.69	70.777	0.000	0.000	0.000	0.000
7.00	25.31	78.601	0.569	0.806	0.405	0.610
8.00	37.26	86.611	0.805	1.035	1.094	1.534
9.00	49.30	95.807	1.008	1.202	2.004	2.659
10.00	61.09	108.412	1.164	1.298	3.095	3.915
11.00	71.63	131.110	1.262	1.316	4.313	5.229
12.00	76.95	180.000	1.296	1.254	5.598	6.521
13.00	71.63	228.890	1.262	1.116	6.882	7.712
14.00	61.09	251.588	1.164	0.913	8.101	8.731
15.00	49.30	264.193	1.008	0.657	9.191	9.520
16.00	37.26	273.389	0.805	0.366	10.102	10.033
17.00	25.31	281.399	0.569	0.061	10.791	10.247
18.00	13.69	289.223	0.000	0.000	11.195	10.253
19.00	2.63	297.491	0.000	0.000	11.195	10.253
20.00	-7.59	306.725	0.000	0.000	11.195	10.253
21.00	-16.54	317.412	0.000	0.000	11.195	10.253
22.00	-23.70	329.937	0.000	0.000	11.195	10.253
23.00	-28.40	344.340	0.000	0.000	11.195	10.253
24.00	-30.05	360.000	0.000	0.000	11.195	10.253

```
Terrestrial Latitude of the site ------------ :   36.500 deg.
Celestial Declination of the Sun of the Sun - :    0.000 deg.
Obstruction Altitude ------------------------ :   15.000 deg.
Solar Constant ------------------------------ :    1.360 kw/m^2.
Horizontal Run of Slope --------------------- :   12.000 unit(s).
Vertical Rise of Slope ---------------------- :    6.000 unit(s).
Compass Azimuth of Slope -------------------- :  135.000 deg.
Displaced Latitude for Slope ---------------- :   16.130 deg.
Displaced Longitude for Slope --------------- :   19.220 deg.
Time of Sunrise for Level Plain of Site ----- :    7.252 hr.
Time of Sunset for Level Plain of Site ------ :   16.748 hr.
Time of Sunrise for Plane of Slope ---------- :    7.252 hr.
Time of Sunset for Plane of Slope ----------- :   16.719 hr.
Diurnal Solar Energy for Level Plain of Site  :    7.907 kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :    8.923 kw-hr/m^2.
```

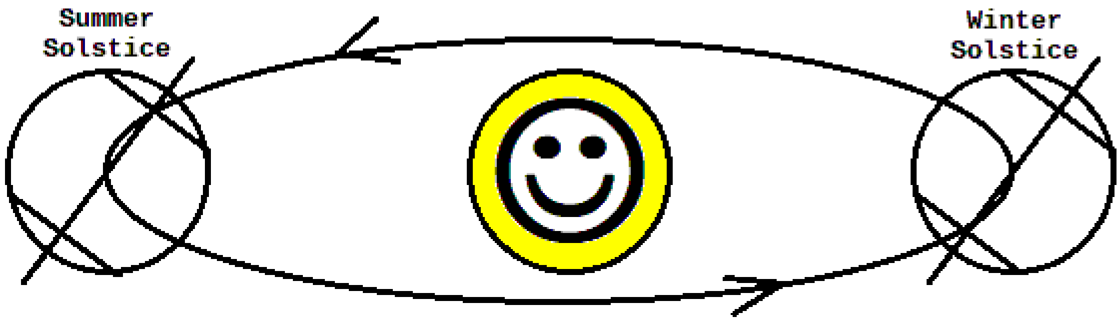
Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	-53.50	0.000	0.000	0.000	0.000	0.000
1.00	-50.94	24.250	0.000	0.000	0.000	0.000
2.00	-44.12	44.146	0.000	0.000	0.000	0.000
3.00	-34.64	59.255	0.000	0.000	0.000	0.000
4.00	-23.70	71.046	0.000	0.000	0.000	0.000
5.00	-12.01	80.944	0.000	0.000	0.000	0.000
6.00	0.00	90.000	0.000	0.000	0.000	0.000
7.00	12.01	99.056	0.000	0.000	0.000	0.000
8.00	23.70	108.954	0.547	0.989	0.337	0.673
9.00	34.64	120.745	0.773	1.176	1.001	1.762
10.00	44.12	135.854	0.947	1.283	1.866	2.999
11.00	50.94	155.750	1.056	1.303	2.873	4.300
12.00	53.50	180.000	1.093	1.234	3.954	5.575
13.00	50.94	204.250	1.056	1.080	5.034	6.739
14.00	44.12	224.146	0.947	0.853	6.041	7.711
15.00	34.64	239.255	0.773	0.568	6.906	8.426
16.00	23.70	251.046	0.547	0.244	7.570	8.835
17.00	12.01	260.944	0.000	0.000	7.907	8.923
18.00	0.00	270.000	0.000	0.000	7.907	8.923
19.00	-12.01	279.056	0.000	0.000	7.907	8.923
20.00	-23.70	288.954	0.000	0.000	7.907	8.923
21.00	-34.64	300.745	0.000	0.000	7.907	8.923
22.00	-44.12	315.854	0.000	0.000	7.907	8.923
23.00	-50.94	335.750	0.000	0.000	7.907	8.923
24.00	-53.50	360.000	0.000	0.000	7.907	8.923

Terrestrial Latitude of the site ------------ :	36.500	deg.
Celestial Declination of the Sun of the Sun - :	-23.450	deg.
Obstruction Altitude ------------------------ :	15.000	deg.
Solar Constant ------------------------------ :	1.420	kw/m^2.
Horizontal Run of Slope --------------------- :	12.000	unit(s).
Vertical Rise of Slope ---------------------- :	6.000	unit(s).
Compass Azimuth of Slope -------------------- :	135.000	deg.
Displaced Latitude for Slope ---------------- :	16.130	deg.
Displaced Longitude for Slope --------------- :	19.220	deg.
Time of Sunrise for Level Plain of Site ----- :	8.814	hr.
Time of Sunset for Level Plain of Site ------ :	15.186	hr.
Time of Sunrise for Plane of Slope ---------- :	8.814	hr.
Time of Sunset for Plane of Slope ----------- :	15.186	hr.
Diurnal Solar Energy for Level Plain of Site :	3.783	kw-hr/m^2.
Diurnal Solar Energy for Slope -------------- :	5.686	kw-hr/m^2.

Time (hr)	Altitude (deg)	Azimuth (deg)	Power Plain	Power Slope	Energy Plain	Energy Slope
0.00	-76.95	0.001	0.000	0.000	0.000	0.000
1.00	-71.63	48.890	0.000	0.000	0.000	0.000
2.00	-61.09	71.589	0.000	0.000	0.000	0.000
3.00	-49.30	84.193	0.000	0.000	0.000	0.000
4.00	-37.26	93.389	0.000	0.000	0.000	0.000
5.00	-25.31	101.399	0.000	0.000	0.000	0.000
6.00	-13.69	109.223	0.000	0.000	0.000	0.000
7.00	-2.63	117.491	0.000	0.000	0.000	0.000
8.00	7.59	126.725	0.000	0.000	0.000	0.000
9.00	16.54	137.412	0.404	0.970	0.072	0.177
10.00	23.70	149.937	0.571	1.072	0.564	1.205
11.00	28.40	164.340	0.675	1.091	1.193	2.294
12.00	30.05	180.000	0.711	1.025	1.892	3.359
13.00	28.40	195.660	0.675	0.878	2.591	4.317
14.00	23.70	210.063	0.571	0.660	3.219	5.091
15.00	16.54	222.588	0.404	0.387	3.712	5.619
16.00	7.59	233.275	0.000	0.000	3.783	5.686
17.00	-2.63	242.509	0.000	0.000	3.783	5.686
18.00	-13.69	250.777	0.000	0.000	3.783	5.686
19.00	-25.31	258.601	0.000	0.000	3.783	5.686
20.00	-37.26	266.611	0.000	0.000	3.783	5.686
21.00	-49.30	275.807	0.000	0.000	3.783	5.686
22.00	-61.09	288.411	0.000	0.000	3.783	5.686
23.00	-71.63	311.110	0.000	0.000	3.783	5.686
24.00	-76.95	359.999	0.000	0.000	3.783	5.686

Chapter IX

The Seasons of the Earth



The Earth orbits about the Sun. The Earth revolves about its polar axis. The Polar axis of the Earth is inclined to the polar axis of the orbit of the Earth. This inclination is known as the obliquity. The obliquity of the Earth varies from 22.1º to 24.5º. The period for the obliquity of the Earth is around 41,000 years. **The current obliquity of the Earth is 23.45º**. The obliquity of the Earth is responsible for the Seasons of the Earth.

The apparent Celestial declination of the Sun varies with the Right Ascension of the Sun. This results in the four Seasons of the Earth. The index is the Spring Equinox. The general equation for the Celestial declination of the Sun is given here as:

[declination] = [obliquity] · [Right Ascension]

Let us now consider the four seasons of the Earth. These arguments apply only to the Temperate Zones.

Seasons of Light

[Vernal Equinox]

At the Vernal Equinox, the ascending Sun is situated on Celestial Equator. The Right Ascension of the Sun is defined as 00:00:00. The declination of the Sun is 00º00’00”. This is the ending of Winter and the beginning of Spring.

[Spring]

During the season of spring, new life is returning to the Earth after a cold and bitter winter. The Sun is ascending in the Celestial Sphere. The temperatures are rising.

[Summer Solstice]

At the Summer Solstice, the ascending Sun has reached its peak altitude in the Celestial Sphere. The Right Ascension of the Sun is defined as 06:00:00. The declination of the Sun is +23º26’10”. This is the ending of Spring and the beginning of Summer. The temperatures are now very warm and still rising.

[Summer]

During the season of summer, the new life of the spring is reproducing itself. The Sun is descending in the Celestial Sphere. The temperatures continue to rise until late summer, after which they either hold steady are begin to descend.

Seasons of Dark

[Autumnal Equinox]

At the Autumnal Equinox, the descending Sun is situated on the Celestial Equator. The Right Ascension of the Sun is defined as 12:00:00. The declination of the Sun is 00°00’00”. This is the ending of Summer and the beginning of Autumn. The temperatures are dropping.

[Autumn]

During the season of Autumn, the life of the Earth is either dying or becoming dormant in preparation for the coming Winter. The Sun is descending in the Celestial Sphere. The temperatures are falling.

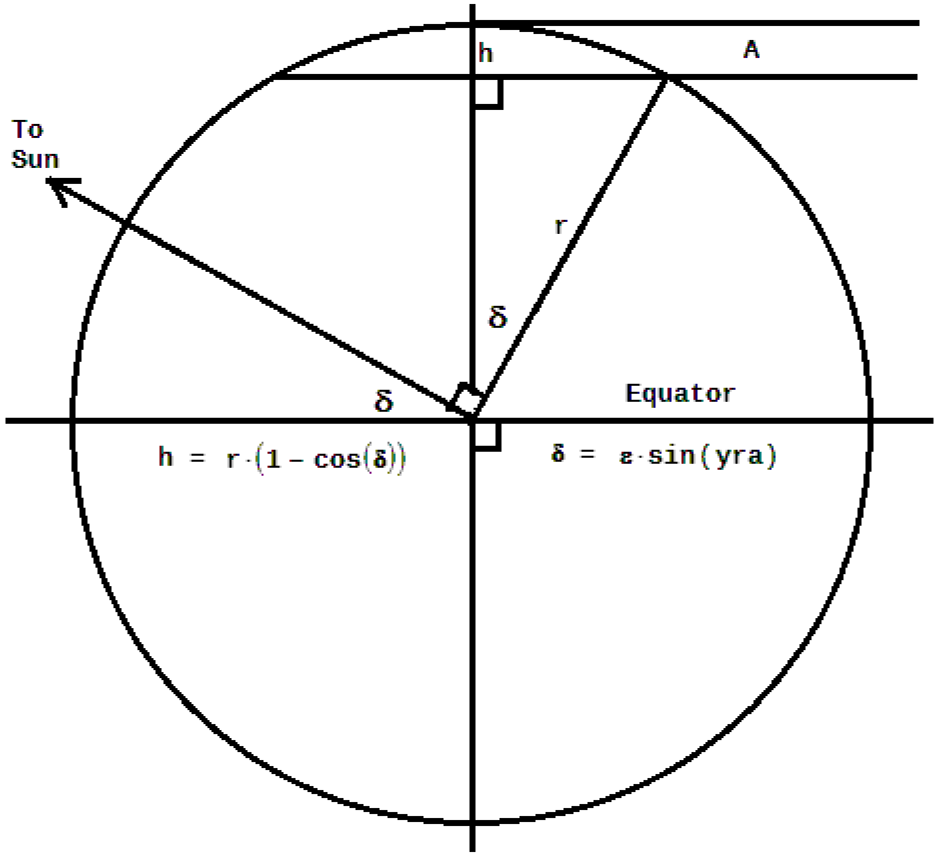
[Winter Solstice]

At the Winter Solstice, the descending Sun has reached its lowest altitude in the Celestial Sphere. The Right Ascension of the Sun is defined as 18:00:00. The declination of the Sun is -23°26’10” This is the ending of Autumn and the beginning of Winter. The temperatures are now cold and still dropping.

[Winter]

During the Winter season, the dying life of the Earth that must die in order to survive has already died. The life of the Earth that must become dormant in order to survive has already become dormant. The temperatures continue to drop until late Winter. After late Winter, the temperatures hold steady or begin to rise. The Sun is ascending in the Celestial Sphere.

Seasons of the Polar Regions



$$A = 2 \cdot h \cdot \pi \cdot r = 2 \cdot (1 - \cos(\delta)) \cdot \pi \cdot r^2$$

$$A = 2 \cdot h \cdot \pi \cdot r = 2 \cdot (1 - \cos(\varepsilon \cdot \sin(yra))) \cdot \pi \cdot r^2$$

$$\frac{dA}{dyra} = \left[\left(\cos\left(\varepsilon \cdot \sin\left(yra - \frac{\pi}{4}\right)\right) - 1\right) + \left(\frac{1 - \cos\left(\varepsilon \cdot \sin\left(\frac{\pi}{2}\right)\right)}{2}\right)\right] \cdot 4 \cdot \pi \cdot r^2$$

The seasons effecting the polar regions and their immediate neighbors is a special case. The polar regions are not of a fixed size. They are constantly expanding and contracting in area. In addition, the rate of change of the area of the polar regions is continuously changing.

Here are the mathematics for the area of the polar regions as well as for the rate of change in the area. The angles are all expressed in radians. **[yra]** represents the year angle from the spring equinox. **[h]** represents the height along the polar axis. **[δ]** represents the declination of the Sun. **[ε]** represents the obliquity of the polar axis. **[r]** represents the radius of the planet. **[A]** represents the area of the polar region.

$$h = r \cdot (1 - \cos(\delta))$$

Angles are all expressed as radians.

$$\delta = \varepsilon \cdot \sin(yra)$$

$$A = 2 \cdot h \cdot \pi \cdot r = 2 \cdot (1 - \cos(\delta)) \cdot \pi \cdot r^2$$

$$A = 2 \cdot h \cdot \pi \cdot r = 2 \cdot (1 - \cos(\varepsilon \cdot \sin(yra))) \cdot \pi \cdot r^2$$

$$\frac{dA}{dyra} = \left[\left(\cos\left(\varepsilon \cdot \sin\left(yra - \frac{\pi}{4}\right)\right) - 1\right) + \left(\frac{1 - \cos\left(\varepsilon \cdot \sin\left(\frac{\pi}{2}\right)\right)}{2}\right)\right] \cdot 4 \cdot \pi \cdot r^2 \quad \sin\left(\frac{\pi}{2}\right) = 1$$

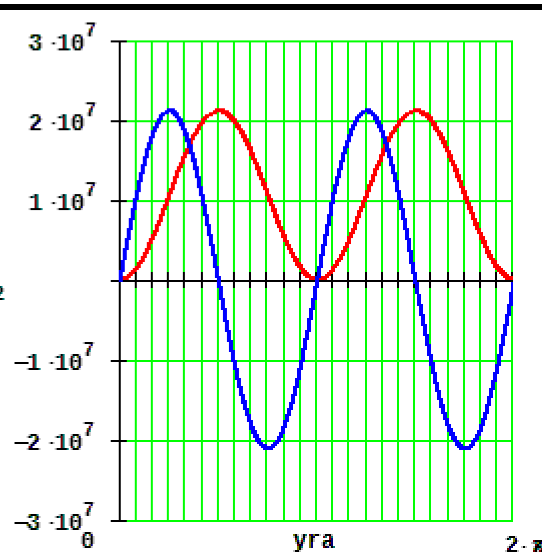
$$\frac{dA}{dyra} = \left[\left(\cos\left(\varepsilon \cdot \sin\left(yra - \frac{\pi}{4}\right)\right) - 1\right) + \left(\frac{1 - \cos(\varepsilon)}{2}\right)\right] \cdot 4 \cdot \pi \cdot r^2$$

$$\varepsilon := 23.45 \cdot \frac{\pi}{180}$$

$$r := 6378$$

$$2 \cdot (1 - \cos(\varepsilon \cdot \sin(yra))) \cdot \pi \cdot r^2$$

$$\left[\left(\cos\left(\varepsilon \cdot \sin\left(yra - \frac{\pi}{4}\right)\right) - 1\right) + \left(\frac{1 - \cos(\varepsilon)}{2}\right)\right] \cdot 4 \cdot \pi \cdot r^2$$



The preceding example is for the polar regions of the Earth. The basic formula for the area is fairly straight-forward. The rate of change of the area of the polar regions is a bit more more involved. The displayed differentiation was found by experimentation. The angles are all expressed in terms of radians.

The polar regions are defined as those areas near the poles that are bathed in continuous light or those areas near the poles that never see the light of day. Each pole alternates during the year between continuous light and continuous dark.

On the surface, the polar regions would appear to have but two seasons. This is not the case. The rate of change of the land area must be taken into consideration.

Here is a point by point break-down of the seasons of the polar regions.

[Vernal-Equinox]

The Vernal-Equinox represents the time of the year at the North Pole that the ascending Sun is lying on the horizon. The Right Ascension of the Sun is 00:00:00 The declination of the Sun is 00º00’00”. The rate of change of the effected land area is at zero.

[Early-Spring]

During Early-Spring, the returning Sun begins to warm up the cold degenerate air of the previous fall and winter. It begins at the North Pole and expands southward. The warming air creates an expanding high pressure ridge over the North Pole that pushes out southward other degenerate arctic air that was laid down during the fall and winter. This causes cold degenerate air to be pushed down from the North into the temperate regions.

[Mid-Spring]

Mid-Spring represents the time of the year that the rate of the expansion of the area of the "Midnight-Sun" has reached its maximum. The Right Ascension of the Sun is 03:00:00 The declination of the Sun is +16º34'54". The positive rate of change of the effected land area is at maximum.

[Late-Spring]

Late-Spring represents the period that the polar region of continuous light is approaching its maximum area. However, the rate of the change is slowing down. No longer is the growing ridge of high arctic pressure of the North Pole pushing bitterly cold air south into the temperate zone.

[Summer-Solstice]

The Summer-Solstice represents the time that the Sun has achieved its greatest declination. The Right Ascension of the Sun is 06:00:00 The declination of the Sun is +23º26'10". The positive rate of change of the effected land area is at zero. The rate of change is poised to reverse and go negative.

[Early-Summer]

During Early-Summer the Sun is just beginning its long descent in the Celestial Sphere. The negative rate of change of the surface area of the polar regions is rising. The temperatures continue slowly rise. The polar ridge of high pressure over the North Pole is firmly in control. It is a period of calmness.

[Mid-Summer]

Mid-Summer represents the time of the year that the rate of the contraction of the area of the “Midnight-Sun” has reached its maximum. The Right Ascension of the Sun is 09:00:00 The declination of the Sun is +16º34’54”. The negative rate of change of the effected land area is at maximum.

[Late-Summer]

During Late-Summer, the potential temperatures reach a maximum and then begin to decline. The thermal ridge of high pressure over the North Pole is still in control. The rate of the contracting area of the “Midnight-Sun” is reducing.

[Autumnal-Equinox]

The Autumnal-Equinox represents the time of the year that the rate of the contraction of the area of the “Midnight-Sun” has reached its Minimum. The period of the “Polar-Night” is poised to begin. The Right Ascension of the Sun is 12:00:00 The declination of the Sun is 00º00’00”. The negative rate of change of the effected land area is at zero. Likewise, the positive rate of change of the effected land area of the beginning of the period of the “Polar-Night” is zero. At the North Pole at the Fall-Equinox, the Sun will be lying on the horizon.

[Early-Fall]

During Early-Fall, the area of the “Polar Night” is beginning to expand. During this period the positive rate of expansion will increase. The expanding area of the “Polar Night” receive no solar radiation. The area will radiate out the thermal radiation into space. There will be a rapid cooling, contracting, and thermal degeneration of the atmosphere in the effected area. Because this is happening over the North Pole, there will be an in-drawing of air from outside the effected area. This will create an "Indian Summer" effect in the neighboring Temperate Zone."

[Mid-Fall]

Mid-Fall represents the time of the year that the positive rate of the expansion of the area of the "Polar-Night" has reached its maximum. The Right Ascension of the Sun is 15:00:00 The declination of the Sun is -16º34'54". The positive rate of change of the effected land area is at maximum. The "Indian Summer" effect of the neighboring Temperate Zone is at its peak.

[Late-Fall]

During Late-Fall, the surface area of the "Polar-Night" is approaching its maximum size. The positive rate of expansion is decreasing. The neighboring Temperate Zone is beginning to assume its natural condition without the "Indian Summer" effect. However, warmer air from the neighboring Temperate Zone is still being in-drawn into the area of the "Polar Night" and degenerated.

[Winter-Solstice]

The Winter-Solstice represents the time that the Sun has achieved its lowest declination. The Right Ascension of the Sun is 18:00:00 The declination of the Sun is -23º26'10". The positive rate of change of the effected land area is at zero. The positive rate of change is poised to reverse and go negative.

[Early-Winter]

During Early-Winter, the Sun begins to return to the polar regions beginning at the North Pole. The negative rate of the contraction of the area of the “Polar-Night” is increasing. The cold degenerated air is expanding and being pushed outward from the North Pole. It is still cold until it encounters the not so cold air of the neighboring Temperate Zone. The interaction produces winter storms. The temperatures continue to fall.

[Mid-Winter]

Mid-Winter represents the time of the year that the negative rate of the expansion of the area of the “Polar-Night” has reached its maximum. The Right Ascension of the Sun is 21:00:00 The declination of the Sun is -16°34’54”. The negative rate of change of the effected land area is at maximum. A reverse of the “Indian Summer” effect of the neighboring Temperate Zone is at its peak. This is potentially the coldest time of the year.

[Late-Winter]

During Late-Winter, the area of the “Polar-Night” is approaching zero. The negative rate of change of area is decreasing. The force of the expulsions of restored Arctic-Air is decreasing. The Earth is preparing for the Spring-Equinox and new life for a new year.

At these 16 accounts illustrate; In the polar regions there are eight seasons. Each of the eight seasons is clearly defined with its own characteristics.

Now let us consider the seasons of the Equatorial Regions.

Seasons of the Tropic Zone

The Tropic Zone consists of all the land and sea lying between +23.45º latitude and -23.45º latitude. The Tropic Zone is a vast area consisting of 38% of the surface of the Earth.

Here are six cases demonstrating the received diurnal radiation of the Sun in the Tropic Zone. Two latitudes are given. Latitude +23.45º represents the outer bound of the Tropic Zone. Latitude 0º represents the inner bound of of the Tropic Zone. Each given latitude is sampled by three declinations. These declinations are +23.45º, 0º, and -23.45º. These three declinations cover the cardinal seasons of the Temperate Zones.

```
Case 1:
Terrestrial Latitude of the site ------------ :   23.450 deg.
Celestial Declination of the Sun of the Sun - :   23.450 deg.
Solar Constant ------------------------------ :    1.330 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    5.277 hr.
Time of Sunset for Level Plain of Site ------ :   18.723 hr.
Diurnal Solar Energy for Level Plain of Site  :   11.231 kw-hr/m^2.

Case 2:
Terrestrial Latitude of the site ------------ :   23.450 deg.
Celestial Declination of the Sun of the Sun - :    0.000 deg.
Solar Constant ------------------------------ :    1.370 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    6.000 hr.
Time of Sunset for Level Plain of Site ------ :   18.000 hr.
Diurnal Solar Energy for Level Plain of Site  :    9.602 kw-hr/m^2.

Case 3:
Terrestrial Latitude of the site ------------ :   23.450 deg.
Celestial Declination of the Sun of the Sun - :  -23.450 deg.
Solar Constant ------------------------------ :    1.420 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    6.723 hr.
Time of Sunset for Level Plain of Site ------ :   17.277 hr.
Diurnal Solar Energy for Level Plain of Site  :    6.594 kw-hr/m^2.
```

```
Case 4:
Terrestrial Latitude of the site ------------ :    0.000 deg.
Celestial Declination of the Sun of the Sun - :   23.450 deg.
Solar Constant ------------------------------ :    1.330 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    6.000 hr.
Time of Sunset for Level Plain of Site ------ :   18.000 hr.
Diurnal Solar Energy for Level Plain of Site  :    9.321 kw-hr/m^2.

Case 5:
Terrestrial Latitude of the site ------------ :    0.000 deg.
Celestial Declination of the Sun of the Sun - :    0.000 deg.
Solar Constant ------------------------------ :    1.370 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    6.000 hr.
Time of Sunset for Level Plain of Site ------ :   18.000 hr.
Diurnal Solar Energy for Level Plain of Site  :   10.466 kw-hr/m^2.

Case 6:
Terrestrial Latitude of the site ------------ :    0.000 deg.
Celestial Declination of the Sun of the Sun - :  -23.450 deg.
Solar Constant ------------------------------ :    1.420 kw/m^2.
Time of Sunrise for Level Plain of Site ----- :    6.000 hr.
Time of Sunset for Level Plain of Site ------ :   18.000 hr.
Diurnal Solar Energy for Level Plain of Site  :    9.952 kw-hr/m^2.
```

At the limit of the Tropic Zone, there are but two seasons. In one season the Sun is descending in the Celestial Sphere and the accumulated diurnal solar energy is falling. In the other season the Sun is ascending in the Celestial Sphere and the accumulated diurnal solar energy is increasing.

The Equator lies at the heart of the Tropic Zone. At the Equator there are four seasons. Beginning at the Vernal-Equinox, the Sun descends in the Celestial Sphere towards the North. Curiously enough, as the Sun is descending towards the North, the accumulated diurnal solar energy levels are rising. Then the Sun ascends again and the accumulated diurnal solar energy levels decline. This half-cycle ends with the Autumnal-Equinox. This process is repeated between the Autumnal-Equinox and the Vernal-Equinox with the Sun descending to the South and then ascending again.

Observe that at the Equator, that the accumulated diurnal solar energy levels are noticeably less than they are outside of the Equator. This is one of those “counter-intuitive” things. In addition, within the Tropic Zone the terms “Spring”, “Summer”, “Fall”, and “Winter” have no meaning.

The data for the solar system for the 62nd edition of the CRC Handbook of Chemistry and Physics was compiled in 1970. In that edition, the longitude of perihelion for the Earth was given as 101.983°.

The Earth and the Sun share the same Celestial Sphere. The Celestial Sphere is mostly represented by the fixed stars. The index is a line drawn from the Sun to the Earth at the time of the Spring Equinox in the Northern Hemisphere on the Earth.

We, living here on the Earth observe the Sun from the opposite angle than the angle from which the Sun views the Earth. For us, here on the Earth, the angle of the longitude of perihelion is advanced or retrogressed by 180°. Thus, the given 101.983° longitude of perihelion is seen by us as either -78.017° or +258.983° with respect to the Sun at the time of the Spring Equinox.

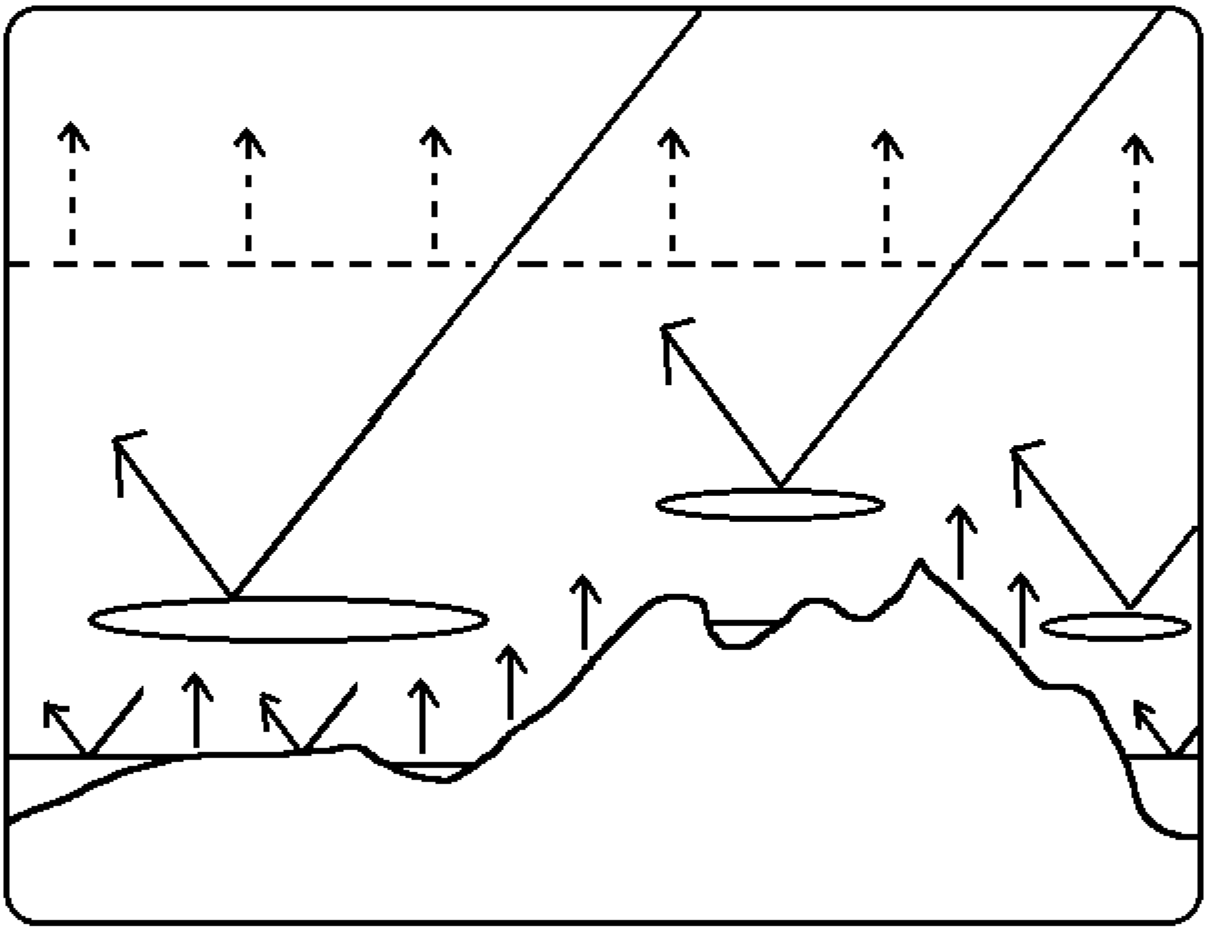
Due to the 25,800 year precession of the equinoxes, the longitude of perihelion will advance by will advance by 0.01395° per year. Today, 52 years later, in the year of 2022, the longitude of perihelion has advanced 0.72558° to 102.70858°. This means that the right ascension angle of the Sun at the time of perihelion will be either -77.291° or +282.709°. This resulting angle will represent the offset angle between the Spring Equinox and perihelion.

The Spring Equinox is the index for the Right Ascension and perihelion is the index for the orbit of the Earth. The former is used to calculate the declination of the Sun and the latter is used to calculate the solar constant.

Chapter X

Solar-Geothermal

The Warming of the Earth



We do not live on the “surface” of the Earth, We live beneath the defacto “surface” of the atmosphere and above the solid or liquid surface. There are typically about 100 kg of atmosphere per square meter.

The Sun warms the air that we breathe. The Sun warms the water that we drink. The Sun warms the solid Earth that we live on. “Earth”, “Wind”, “Water”, and the Sun is “Fire”. These are the four ancient elements comprising all of the natural World.

This discussion of the matter of Solar-Geothermal energy must begin with the albedo of the Earth. The albedo of the Earth represents the percentage of the radiation of the Sun that is directly reflected back into space without being absorbed. The mean albedo for the Earth is around 35%. This leaves 65% to be absorbed by the Earth and converted to heat as solar-geothermal energy. The solar-geothermal energy is then re-radiated from the top of the atmosphere back out into space as thermal radiation in the deep infrared. The local albedo will vary from site to site.

As the Sun’s rays pass through the atmosphere, a small portion of the energy is absorbed. This absorbed energy causes a rise in the temperature of the atmosphere. This effect is most intense when the Sun is at a lower altitude due to there being more atmosphere in the path of travel. This atmospheric heat will be passed on to the underlying water and solid Earth.

When the Sun’s rays hit the waters of the Earth; a portion is reflected and the remainder is absorbed causing the water to warm. A small portion of the portion that is reflected is absorbed by the atmosphere resulting in a warming of the air. The remainder is radiated out into space as part of the albedo. The warmed water in turn passes its heat on to the overlying air and the solid earth beneath.

The Sun’s rays that make it to the surface of the solid Earth will either be reflected or absorbed. The portion that is reflected will be partially absorbed by the atmosphere resulting in more heating of the air, and the remainder will act as a component of the albedo. The portion that is absorbed will cause an intense local heating of the Earth; and a more general, but less intense heating of the whole body of the Earth.

The heated atmosphere, the heated waters, and the heated Earth are the three components of the solar-geothermal energy. These three components are always exchanging heat energy either by direct physical contact of by close proximity thermal radiation. The Earth beneath the top of the atmosphere is always and continuously recycling solar-geothermal heat energy. At the very top of the atmosphere, the thermo-radiant energy of the Earth will be truly reflective of the absorbed solar radiation. What comes in must go out, “… spinning wheel, spinning round ….” r

The mean solar constant for above the atmosphere is 1.373 kw per square meter. This translates to an equilibrium temperature of 394 K, 121ºC or 250ºF. 65% of this energy is absorbed by the Earth, including, the atmosphere, the waters, and the solid Earth. The mean absorbed radiation comes to 892 Watts per square meter. In general, this absorbed radiation will be sent back out into space at 25% of the intensity of the incoming radiation. This is because the surface of a sphere is always 4 times greater the area of a circle for the same radius. Thus, the mean out-going radiation for the Earth will be 223 Watts per square meter of therm-radiant energy in the deep infrared. The adjusted equilibrium therm-radiant out-going temperature will be 250K, -23ºC, or -9ºF. The reality must by necessity fall between the two extremes.

The Solar-Geothermal Balance

The solar-geothermal balance is the accumulated difference in solar energy received from the Sun versus geothermal radiant energy lost to space. This balance comes in two forms. One form is the diurnal balance and the other form is the annual balance

Now let us consider a bit of natural philosophy pertaining to the Solar-Geothermal balance.

It is a common misconception to imagine that our bodies are warmed by the Sun or cooled by the lack thereof. The truth is that the Sun warms the environment that our flesh in in contact with. This statement must be qualified. To a degree the Sun does warm our bodies. The direct rays Sun warms the surface of the skin which then to a small degree warms our internal organs. Likewise, to a small degree, the lack of sunlight permits the excess heat to escape which results in cooling. However, under many of the environmental conditions found on the Earth, this can be an unstable process. Only in the environment in which our bodies originated may this minor process of heating and cooling be considered stable.

The garments that we wear have their origins in aiding the body’s internal temperature regulatory systems. They garments’ create a localized thermal environment akin to the general physical environment that our bodies originated in. They are for intended for use when we are living outside of our natural habitat. The houses that we dwell in also act as garments. However, neither garments nor houses were ever intended to become cultural fetishes by any name!

The use of garments can be a boon in their place, but they also can become life threatening liabilities when employed outside of their place. They can result in the overheating of the natural body, prevent exposure to needed sunlight, or cause an immune response by attempting to reject the garments resulting in apparent sickness!

The Diurnal Solar-Geothermal Energy Balance

Now let examine the mathematics of the diurnal solar-geothermal energy balance. We will index this balance to midnight. The period will be from midnight to midnight, i.e. from nadir to nadir.

The process will involve the thermal mass of the local atmosphere, the first 12 inches of the surface water, and the first 6 inches of the soil. Even though these three are integrated to the whole of the Earth, it is in these three areas that the diurnal solar-geothermal will be most pronounced.

The mass involved will store the incoming solar radiation by day while continually radiating it out as thermo-radiant energy. The latter will be computed as the prorating of the total accumulated solar energy for the day less the albedo. The incoming accumulated energy and the out-going therm-radiant energy will be calculated. The two will be arithmetically compared.

Observe that these are guide lines and are not precise calculations. It is the critical points in the calculations that matter.

Here is the step by step order of the calculations.

[01]: [latitude]

This is the first of the three given variables. [latitude] represents the Terrestrial latitude of the site with respect to the Terrestrial Equator. [latitude] is expressed in terms of degrees.

[02]: [declination]

This is the second of the three given variables. [declination] represents the Celestial Declination of the Sun with respect to the Celestial Equator. [declination] is expressed in terms of degrees.

[03]: [sol]

This is the third of the three given variables. [sol] represents the solar constant less the albedo of the Earth. [sol] is expressed in terms of kilowatts per square meter.

[04]: [b]

This is a calculated variable representing the Terrestrial latitude of the site with respect to the Terrestrial North Pole. It is expressed in terms of radians.

$$b = (90 - latitude) \cdot \frac{\pi}{180}$$

[05]: [c]

This is a calculated variable representing the Celestial declination of the Sun with respect to the Celestial North Pole. It is expressed in terms of radians.

$$c = (90 - declination) \cdot \frac{\pi}{180}$$

[06]: [p]

Since both [b] and [c] are derived directly from the given constants of latitude and declination, [p] acts as an extended constant representing the products of the respective cosines of [b] and [c]. [p] is always paired with [q]. [p] is utilized for clarity in the calculations.

$$p = \cos(b) \cdot \cos(c)$$

[07]: [q]

Since both [b] and [c] are derived directly from the given constants of latitude and declination, [q] acts as an extended constant representing the products of the respective sines of [b] and [c]. [q] is always paired with [p]. [q] is utilized for clarity in the calculations.

[08]: [d]

[d] is used here as a variable representing the positive time of sunrise and sunset with respect to the zenith of the Sun. In this particular case, it is first utilized as a test value with an unlimited range, and then it is recycled as an angle with a limited range. It is expressed in terms of radians.

$$d = \frac{(\theta - p)}{q} \quad \begin{cases} d = \pi & d \le -1 \\ d = \mathrm{acos}(d) & -1 < d < 1 \\ d = 0 & 1 \le d \end{cases}$$

[09]: [Σen]

The variable [Σen] represents for the sum of the energy of the Sun for the given day. It is given in terms of kilowatt-hours.

$$\Sigma en = \frac{24 \cdot sol}{\pi} \cdot (p \cdot d + q \cdot \sin(d))$$

[10]: [δen]

The variable [δen] represents the uniform prorated rate of received solar radiation for the 24 hour period from midnight to midnight. It is expressed in terms of kilowatt-hours per hour in order to compare it with the kilowatt-hours of accumulated energy and the kilowatt-hours/hour of power. [δen] represents the mean solar-geothermal energy rate of loss back to space.

$$\delta en = \frac{\Sigma en}{24} = \frac{sol}{\pi} \cdot (p \cdot d + q \cdot \sin(d))$$

[11]: [hour]

This is a given variable representing the conventional hour of the day beginning at midnight. It is employed for sampling during the course of a given day. It must be converted to radians and then further converted to the spherical trigonometric reference for further processing.

[12]: [α]

[α] represents the time of the day from midnight (nadir) to the following midnight (nadir) with respect to the time of noon (zenith). It is expressed in terms of radians. Before zenith has a negative [-] value and past zenith has a positive [+] value.

$$\alpha = (hra - 12) \cdot \frac{\pi}{12}$$

[13]: [en]

[en] represents the accumulated absorbed solar energy from nadir to the following nadir for the given time of the day with respect to the same.

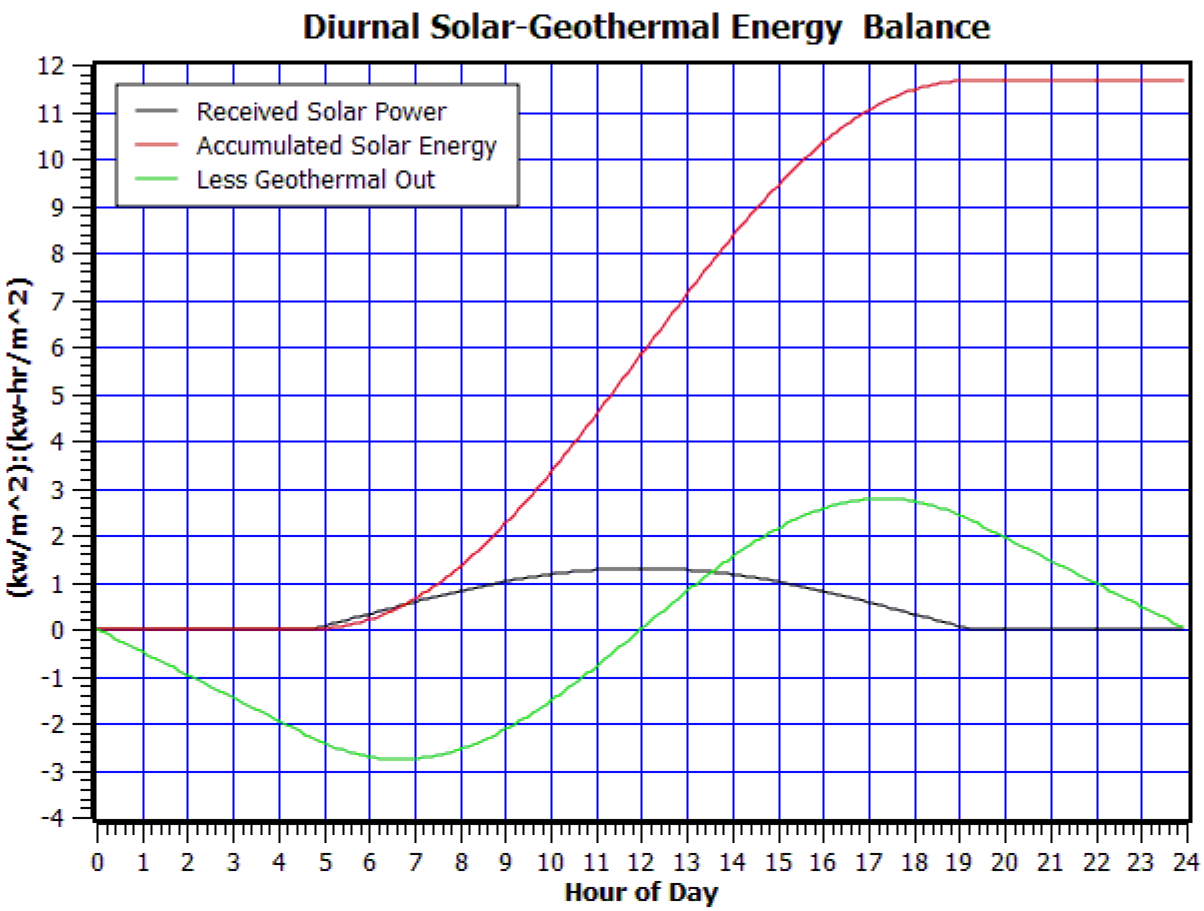
$$\begin{cases} en = 0 & \alpha \le -d \\ en = \frac{12 \cdot sol}{\pi} \cdot (p \cdot d + q \cdot \sin(d)) + \frac{\Sigma en}{2} & -d < \alpha < d \\ en = \Sigma en & d \le \alpha \end{cases}$$

[14]: [bal]

[bal] represents the balance or the difference between the rate of absorption of solar energy and the pro-rated release of the absorbed solar energy. Technically and logically, the latter would reflect the absorbed solar energy of the previous day.

$$bal = en - \delta en \cdot hour$$

Here we have a graph of the solar-geothermal balance equations. The situation is for the second of two cloudless days around the time of the Summer Solstice on a blackened volcanic desert at 36.5° North latitude. The sinusoidal green trace represents the solar-geothermal balance. Observe the time lag. This balance closely reflects the diurnal heating and cooling of the site.



The following scenarios are according to this graph:

At midnight the Earth is releasing therm-radiant energy but is not receiving any solar energy to replace it with. This causes the Earth to cool.

At sunrise the Earth is still losing heat heat energy, but the Sun is returning.

There is now a time lag between Sunrise and the time that the Earth stops cooling down. During this period the Sun is warm, but the air overlying the Earth is still cooling.

About an hour to two hours after sunset the solar-geothermal balance bottoms out. This represents the “chill” of the morning.

After the “chill” of the morning, the solar-geothermal balance is rising. The Earth is warming up. The Sun is warm. However, the solar-geothermal balance is still less than zero. This continues until noon when the Sun stands at zenith.

At noon, when the Sun stands at zenith, the solar-geothermal balance is at zero. The Earth is now poised to absorb new energy.

During the afternoon the Earth is absorbing surplus energy. As this surplus energy is absorbed into the body of the Earth, it warms the overlying air. This continues until about an hour or two before sunset.

About an hour or two before sunset, the solar-geothermal balance reaches a maximum. The Sun is near setting, but the temperatures have reached their highest point for the day. This is commonly referred to as the “heat” of the day.

Between the “heat” of the day and sunset, the solar-geothermal balance is falling. However, it is still above its mean. The Earth is radiating out its warmth as the ultimate source of that warmth, the Sun, is setting.

Between sunset and midnight, the solar-geothermal balance is falling, but is still above the mean value. Until midnight, the Earth is still radiating out therm-radiant energy. This creates warm Summer evenings.

On the surface it would seem like calculating the time lag would be a simple matter. Sadly, that is not the case.

To calculate the exact time lag involves what is sometimes referred to as a “transcendental” equation. They can be very complicated and can never be solved perfectly, even symbolically. They occur when a conflict arises between an orthogonal coordinate equation and a polar coordinate equation.

All such equations have to be solved as one-off piecemeal equations. This is commonly done by creating a table using small increments. It is also done as the product or sum an infinite series of nested calculations.

For universal situations, we have the solution for π, the infinite series for the sines, and the infinite series for the arc-sine. In the first case we attempt to resolve π as close as possible. In the latter cases, we use the resolved decimal value of π to create our trigonometric tables. In all of these cases, the value of π and the trigonometric tables are commonly used by all. Unfortunately, the requisite tables for this problem are unique for each and every situation. Here are two partial tables for the preceding example. Observe the 1.9 hour lag.

Hour	Power	Energy	Balance
4.70	0.000	0.000	-2.283
4.80	0.012	0.000	-2.332
4.90	0.036	0.003	-2.378
5.00	0.061	0.008	-2.422
5.10	0.086	0.015	-2.463
5.20	0.111	0.025	-2.502
5.30	0.136	0.037	-2.538
5.40	0.161	0.052	-2.572
5.50	0.187	0.069	-2.603
5.60	0.212	0.089	-2.631
5.70	0.238	0.112	-2.658
5.80	0.263	0.137	-2.681
5.90	0.289	0.165	-2.702
6.00	0.315	0.195	-2.720
6.10	0.341	0.227	-2.736
6.20	0.366	0.263	-2.749
6.30	0.392	0.301	-2.760
6.40	0.417	0.341	-2.768
6.50	0.443	0.384	-2.774
6.60	0.468	0.430	-2.777
6.70	0.494	0.478	-2.777

Hour	Power	Energy	Balance
17.30	0.494	11.182	2.777
17.40	0.468	11.231	2.777
17.50	0.443	11.276	2.774
17.60	0.417	11.319	2.768
17.70	0.392	11.360	2.760
17.80	0.366	11.397	2.749
17.90	0.341	11.433	2.736
18.00	0.315	11.466	2.720
18.10	0.289	11.496	2.702
18.20	0.263	11.523	2.681
18.30	0.238	11.548	2.658
18.40	0.212	11.571	2.631
18.50	0.187	11.591	2.603
18.60	0.161	11.608	2.572
18.70	0.136	11.623	2.538
18.80	0.111	11.636	2.502
18.90	0.086	11.645	2.463
19.00	0.061	11.653	2.422
19.10	0.036	11.658	2.378
19.20	0.012	11.660	2.332
19.30	0.000	11.660	2.283

The Annual Solar-Geothermal Energy Balance

So much for the diurnal solar-geothermal energy balance. The diurnal solar-geothermal energy balance primarily effects the immediate surface of the Earth due to its rapid shifts.

The annual solar-geothermal energy balance is a different matter altogether. It has considerably more time to penetrate into the body of the Earth and the seas.

In order to demonstrate the effect and to simplify the equations, a circular orbit of 360 days will be assumed. A mean absorbed solar radiation of 0.940 kilowatts per square meter will employed throughout. The obliquity will be assigned a value of 23.45º. The equations will be indexed to the Vernal Equinox. This will not be intended as a precision demonstration. Such precision is impossible.

In truth, for this section, the equations will not be shown. The components for the equations have already been given. From those components the following graph was created for our imaginary year of 360 days for a Terrestrial latitude of N 36.5º. An imaginary circular orbit is assumed. An absorbed solar constant of 0.94 kilowatts per square meter is assumed. We will consider the traces one by one.

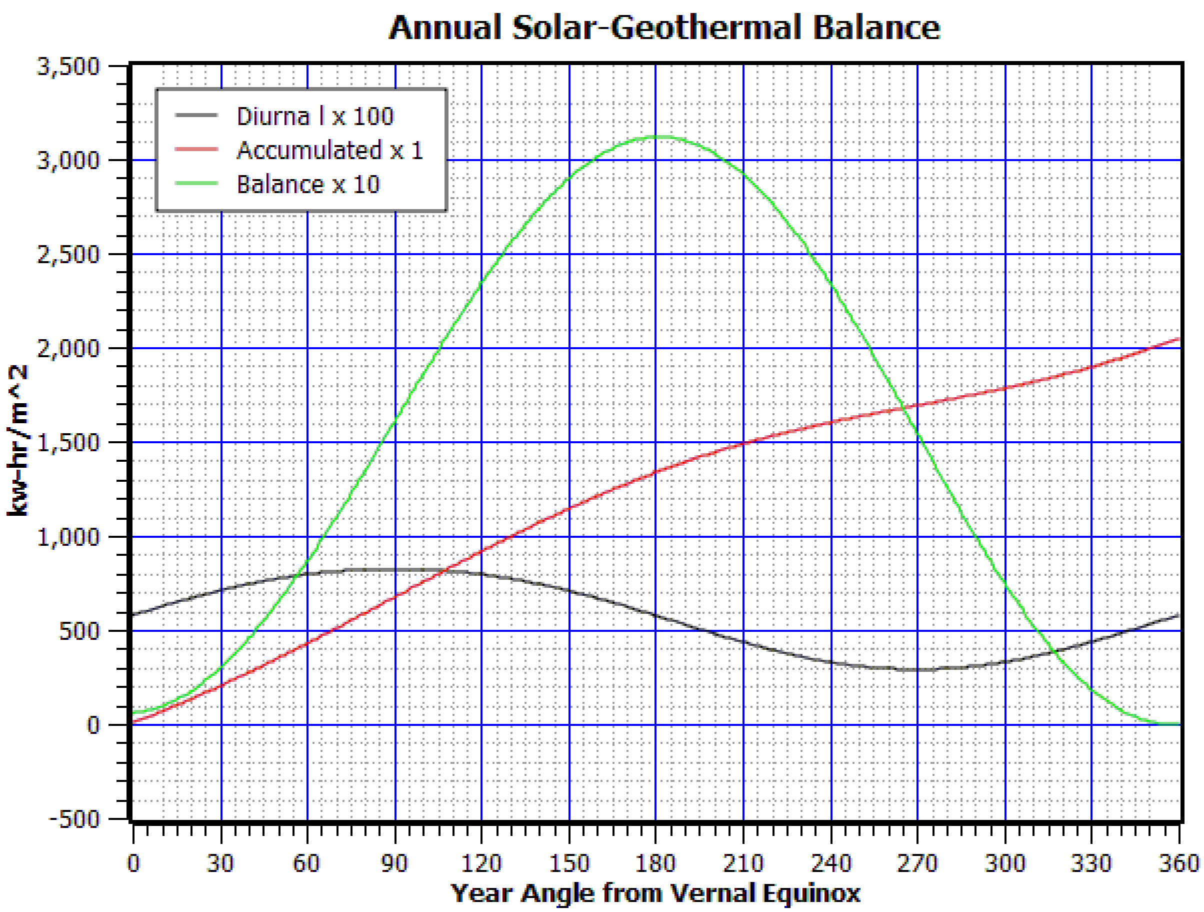
The independent variable is the year angle. In the graph it is expressed in terms of degrees. This is the reason that an imaginary year of 360 days was chosen. The year angle begins at the time of the Vernal Equinox. From the year angle, the required declination of the Sun was calculated.

[dec = 23.45º · sin(yra)]

The gray trace represents the accumulated diurnal energy that has been absorbed into the surface of the Earth for the given day of the year. In order to compare it to the total accumulated energy for the year, it has been multiplied by a factor of 100.

The red trace represents the sequential sum of the accumulated absorbed diurnal solar energy from the Vernal-Equinox.

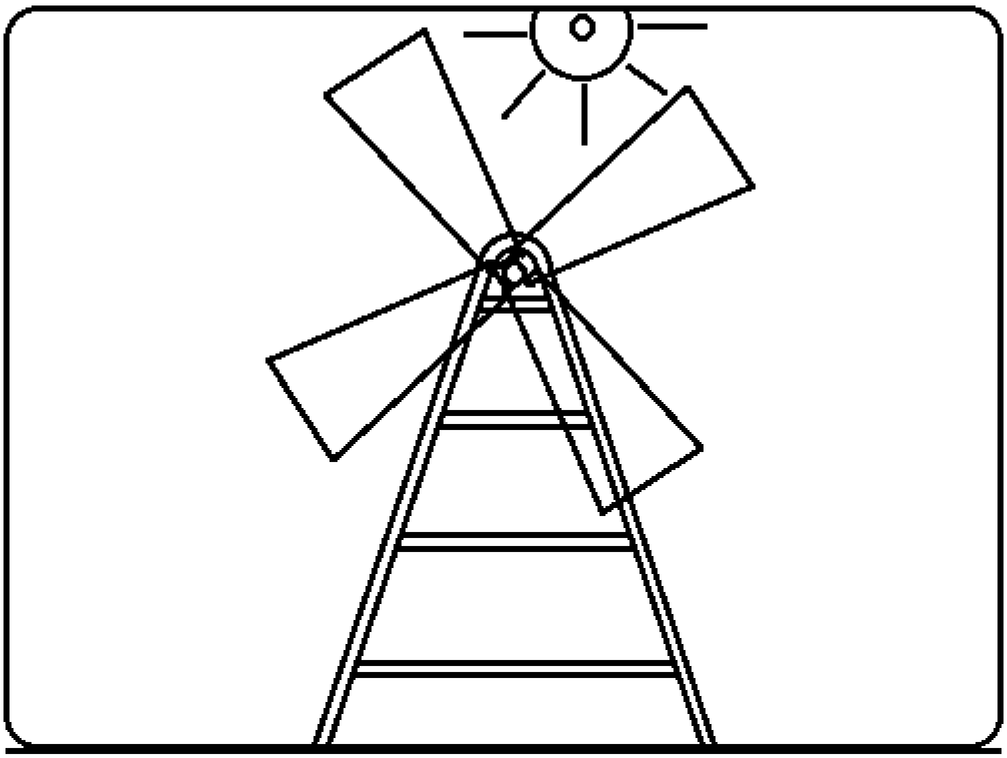
The green trace represents the solar-geothermal balance between the annual accumulation less the pro-rated loss due to therm-radiant out-going energy. The latter is determined by dividing out the accumulated energy by the number of days in the year in order to produce a mean loss to space. Observe that the Earth is coldest at the time of the Vernal-Equinox and hottest at the time of the Autumnal-Equinox.



This concludes this chapter on the solar-geothermal balance. Now let us move on to the Winds of the Earth.

Chapter XI

The Winds of The Earth



The Solar Energy and Wind Energy Connection

Earth, Air, water, Fire.

Earth, Air, and Water are all descended out of Fire.

Fire is the Sun.

Chapter X discussed the Fire acting on the Earth.

This Chapter will discuss the Fire acting on the Air.

In General:

Natural Conservation of energy

The Earth revolves about its polar axis from West to East. This results in an apparent motion of the Sun from East to West.

As the Earth revolves about its polar axis from West to East; the mountains, the valleys, the hills, the trees, the hollows, and the waves of the sea all carry the atmosphere with them. Thus, due to natural friction, the atmosphere will also rotate from West to East about the polar axis of the Earth. However, there will be no apparent rotation due the rotation occurring at the same angular velocity as the surface of the Earth.

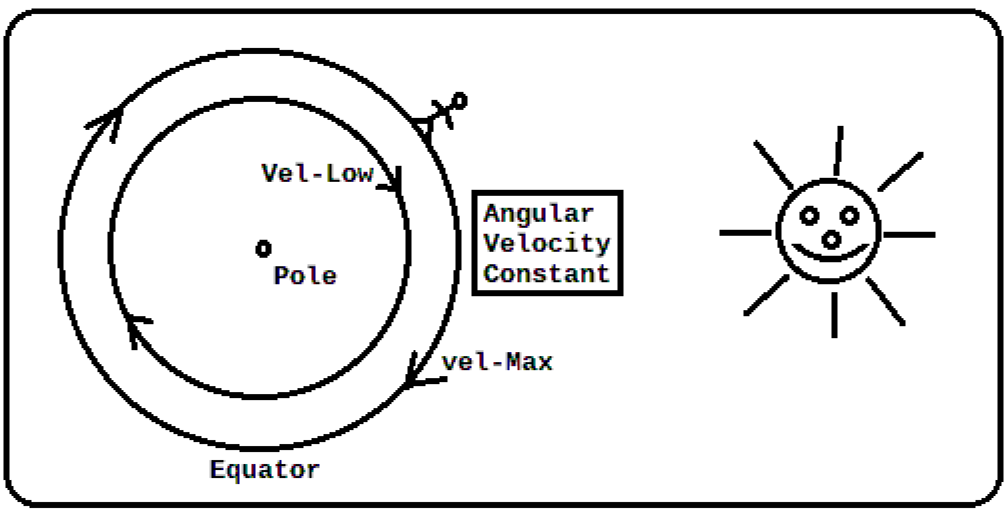
The highest potential absolute velocity of the atmosphere is at the equator. The lowest potential absolute velocity of the atmosphere is at the poles. At the equator, the potential absolute velocity is around 1,050 mph (1,670 kph). At the poles the potential absolute velocity is zero.

An observer on the surface of the Earth would never experience the absolute potential velocity of the atmosphere. This is because the observer will always be subject to the same potential absolute velocity as the immediately surrounding atmosphere.

However, conservation of energy applies. The difference in the absolute potential velocities of the atmosphere at varying latitudes will cause a system of harmonic vortices to form between the Equator and the Poles. These harmonic vortices will serve to reduce the absolute velocity without any conflicts. The Equator will always be in control of the system.

As the system of harmonic vortices moves out from the Equator, there will be latitudes where the absolute velocities will be less than the absolute potential velocities. There will also be latitudes where the absolute velocities will be greater than the absolute potential velocities. This effect will appear as a series of diminishing sine waves whose net value is zero. However; To the observer on the surface of the Earth, these variations will appear as comparatively mild “trade winds.” These “trade winds” will also drive the surface ocean currents. These combination of these two items are at the very heart transoceanic trade. The conservation of energy comes in to play where two neighboring vortices meet with identical velocities and vectors, howbeit opposite rotations.

Whenever a “trade wind” appears to blow towards the Poles, it is really blowing to the East of the pole. This is because there is already a dominant Eastward motion. Whenever a “trade wind” appears to blow towards the Equator, it is really blowing to the West of a position due South. This latter case is simply an inversion of the former case. It is all about real versus apparent vectors in space.

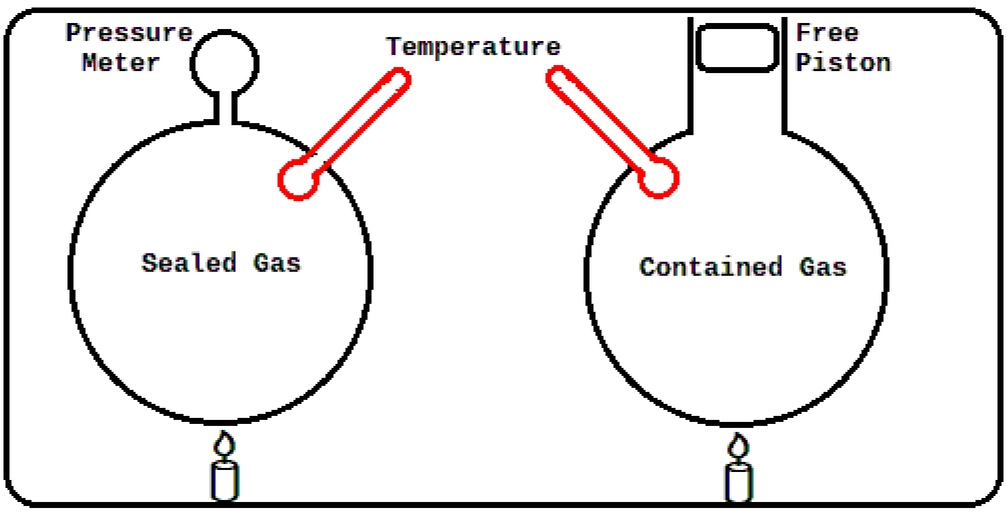


A quick word about “trade winds.” The trade winds blow around the vortices. The established trade winds touch on opposing continents. These are the shipping lanes. There are some vortices that never come near land. Many a sailing vessel has become trapped in these deadly vortices with the sails set, the crew expired, and the rudder locked as a “Ghost Ship” to sail in a circle forever.

. Gas Thermodynamics

The Carnot Efficiency Equation

In the early days of experimentation with gases, certain observations were made. In was observed that the pressure of a sealed gas decreased linearly with a decrease in temperature. Likewise, it was observed that the volume of a gas under conditions that the gas was free to expand decreased linearly with a decrease in temperature. These two observations were theoretically extended to the limit and the result was named “absolute zero.”



In the Celsius scale of Temperatures, absolute zero is -273.15°C. The absolute temperatures using the Celsius Scale are referred to as “Kelvin” [k] without the mention or symbol of “degree” [°].

Here are some of the observed mathematical relationships:

[Pressure1/Pressure2 = Temperature1/Temperature2]

[Volume1/Volume2 = Temperature1/Temperature2]

For these relationships, the gas is referred to as an “ideal” gas. An ideal gas is considered infinitely compressible as the density approaches zero. This is not realistic, but this is what will be used in the arguments.

The Carnot efficiency equation is commonly applied to heat engines that depend on an expanding gas for their operation. Technically, it refers to the excess component of the volumetric expansion of an ideal gas due to an application of heat. Both temperatures are always expressed on the absolute temperature scale. The Kelvin temperature, an extension of the Celsius Scale is most often applied. Here is the Carnot efficiency equation:

[Efficiency = 1 - Temp_low/Temp_high]

In this equation; The low temperature represents the base volume condition of the gas and the high temperature represents the expanded volume of the gas. The work is done only by the component of the gas that was expanded due to the addition of heat energy. However, the entire mass of the air had to be heated from the low temperature to the high temperature in order to acquire the excess volume that is doing the work. Technically; The low temperature divided by the high temperature represents the inefficiency. The efficiency is found by subtracting the inefficiency from one.

The atmosphere of the Earth acts as the fluid gas of a heat engine under the energizing rays of the Sun, or the thermo-radiant energy of the Earth. However; Unlike a mechanical heat engine, the atmosphere is not contained. The atmosphere is free. As soon as the air is heated it expands and moves out. There is never any chance of significant efficiency. It is an inefficient system. This inefficient heat-engine system is what drives the dynamic winds of the Earth.

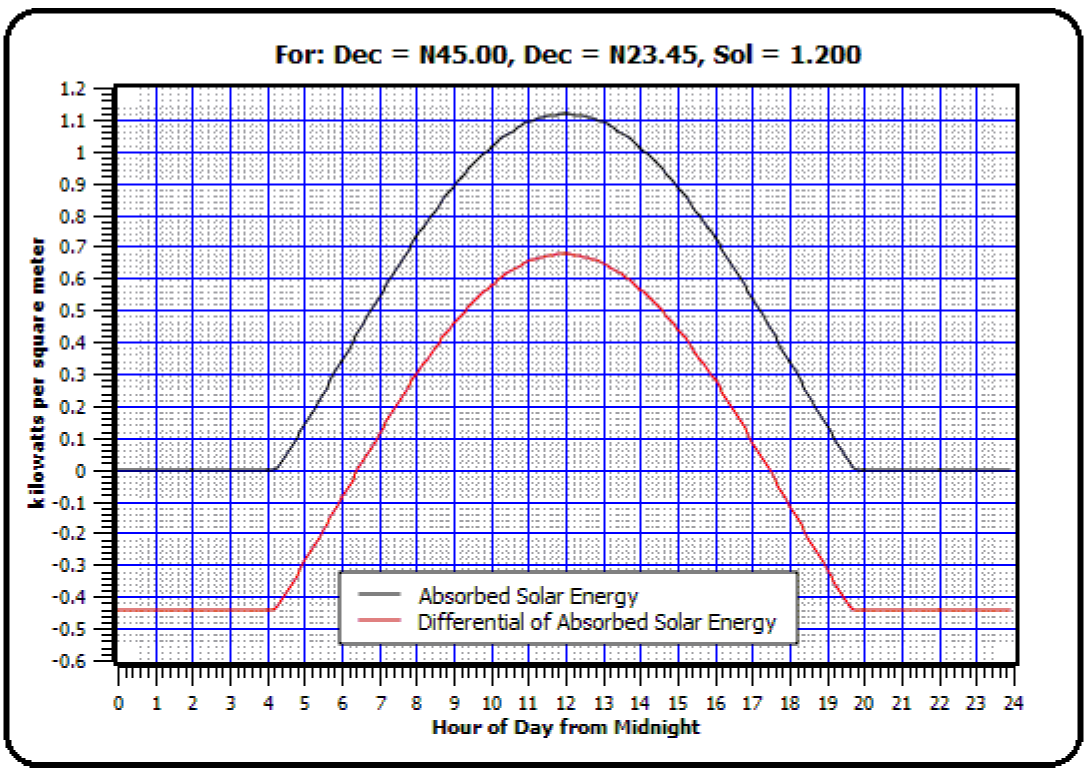
As a section of the Earth heats up, the expanded air will form a “ridge” above the surrounding air at the top of the atmosphere. Due to gravity, this “ridge” will “roll-off” seeking an an equilibrium with the surrounding atmosphere. This “roll-off” will appear as the high-altitude winds. Some of it will be passed on to the lower altitudes.

The Longitudinal East↔West Component

Diurnal Wind Variations

As the Earth rotates about its polar axis under the Sun, it is warmed the by the energetic rays of the Sun. The atmosphere which passes most of the radiation is warmed very little. The dark opaque Earth below which absorbs most of the radiation is greatly warmed. The warmed Earth in turn heats the overlying atmosphere. The heated atmosphere expands upwards and outwards. The overlying ridge of higher pressure then collapses “downhill”, producing an east-west force of wind.

Take a look at the following graph. It is typical of a clear sunny day. This graph also provides an indication for the solution to a bothersome problem from the previous chapter.



The upper gray trace of the graph represents the absorbed radiation of the Sun while the Sun is above the horizon. It is virtually identical to the power. The only difference is an allowance for the local albedo. In this case, it is a clear sunny day with a local albedo of around 0.1 to 0.2. During the night the only radiation available is the thermo-radiant energy of the Earth. The overlying atmosphere is heated by a balance of the two. Thus, during the night a value of zero is assigned.

The lower red trace represents an approximation of the diurnal solar-geothermal balance. This approximation was obtained by first calculating the accumulated energy in 0.1 hour increments throughout the day. Then the accumulated energy calculations were done a second time for the same, but 0.1 hour advanced. This is more or less according to the rules of differentiation. Then the first set of calculations were subtracted from the second set of calculations. Finally, the preceding result was multiplied by 10 to index it to the hour.

$$[a \cdot (x+h)^n + b] \quad h \to 0$$

This approximation as shown in the graph yielded another tentative solution to the problem of the differential of the diurnal solar-geothermal balance from the preceding chapter. However, the solution cannot be expressed as a simple equation.

The tentative solution is surprisingly simple. For any given time of the day; simply subtract the mean out-going thermo-radiant energy of the Earth from the “positive” incoming solar radiation. In the event that the Sun is below the horizon, the incoming solar radiation is assigned a value of zero.

The differential in question will determine the direction and the intensity of the atmospheric slope.

The Sun appears to revolve about the Earth from East to West. Points East have identical events occurring at an earlier time. Points West have identical events occurring an a later time. When an advance in time yields a higher energy level, that higher energy level has already occurred to the East. In such a case, the resultant force driving the wind must be from East to West. Thus a positive differential will represent the force driving an East wind. Alternately, a negative differential will represent the force driving a West wind.

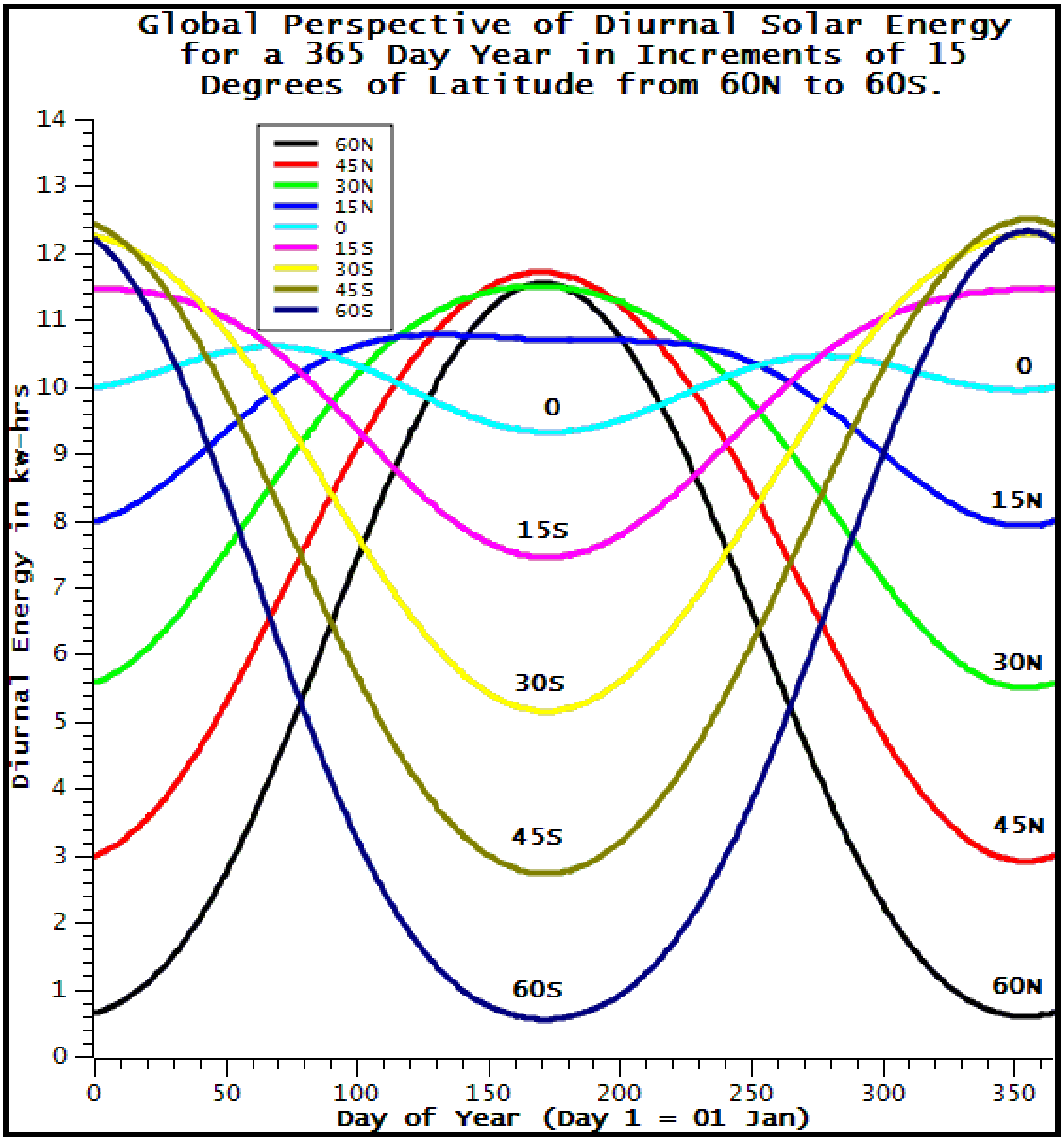
This may be seen on a bright sunny day. During midday the wind often picks up violently. Alternately, at night the wind is more steady. The latter is driven solely by the out-going therm-radiant energy of the Earth itself. Sailors on the high seas have long noted this phenomenon.

Observe that this effect may join or oppose the natural winds of the Earth due to the varying rotational velocities that result from different latitudes. Where they join the winds may become exceedingly great. Where they oppose; a calm and a standing ridge of high pressure may prevail. In both cases, the effect will reverse twice every 24 hours.

Under dense clouds, there may be no wind at all. The out-going thermo-radiant energy may balance with the incoming solar radiation.

Let us now move on to the North-South latitudinal component. We will begin by taking a look at the global perspective of the received solar energy. The following graph tells much of the story. The variations of the solar constant have been included. The latitude sampling increment is 15º.

The Latitudinal North-South Component



Annual Solar Energy per Square Meter

60N = 2091 Kilowatt-Hours per Year.
45N = 2713 Kilowatt-Hours per Year.
30N = 3226 Kilowatt-Hours per Year.
15N = 3557 Kilowatt-Hours per Year.
0 = 3670 Kilowatt-Hours per Year.
15S = 3562 Kilowatt-Hours per Year.
30S = 3226 Kilowatt-Hours per Year.
45S = 2713 Kilowatt-Hours per Year.
60S = 2091 Kilowatt-Hours per Year.

Let us consider two basic situations as indicated by the preceding graph. This is the situation for the temperate zone and the equatorial zone. The data defining the polar zone is not supplied. However, the polar zone appears to be continuation of the temperate zone.

Situation 1: This is the case for an observer in the temperate zone between 23.45º and 66.55º. The hemisphere is not significant.

At the time of the Summer-Solstice, the diurnal radiation is virtually the same throughout the zone. In such a case, there can be no differential either pole-ward or equator-ward for the observer. There is no incentive for the development of a North or a South meteorological force. All is calm.

At the times of the Vernal-Equinox and the Autumnal-Equinox, There is a clear separation of the diurnal energy of the Sun with respect to the latitude. The lower latitudes receive a greater amount of energy and the higher latitudes receive a lesser amount of energy. The spread appears to be generally in proportion to the latitude.

At the time of the Winter-Solstice, the separation of the diurnal energy of the Sun with respect to the latitude has reached its maximum. The spread is still more or less in proportion to the latitude.

With the exception of the Summer-Solstice, this situation will tend to drive the wind pole-ward. However; We live in a packed Universe and on a packed Earth. When the air attempts to rush into the polar regions from all sides it encounters itself, but still the force is on. Something has to give.

The Earth rotates from West to East. The heated atmosphere rotates from West to East. The more energetic lower altitudes rotate at a higher velocity than the higher latitudes. Thus, the pole-ward push will “miss” the pole to the East. The incoming air will form a

"Polar-Vortex" about the Pole. In the Northern Hemisphere, this will result in a Northeast feed vector (SW wind), feeding a counter-clockwise "Polar-Vortex." In the Southern Hemisphere, this will result in a Southeast feed vector (NW wind), feeding a clockwise "Polar-Vortex."

This effect in the Temperate Zones will, when combined with the daytime easterly flow of air due to the diurnal energies, result in a generally easterly flow of air (West Wind).

This same effect in the Temperate Zones will, when countered by the nighttime westerly flow of air due to the diurnal energies, result in a general nighttime calm.

Generally Speaking; As the daytime diurnal energy of the Sun is declining due to the progression of the seasons, the daytime strength of the wind will be increasing. Thus, the inhabitants of the Temperate zones would be well advised to employ both the means of employing solar energy and the means of employing wind energy that is available and at their disposal for their time and place.

Situation 2: This is the case for an observer in the Equatorial zone between 23.45° North Latitude and 23.45° South Latitude.

In the Equatorial Zone, twice a year, the diurnal energy differential will go to zero. This will occur about the time of the respective equinoxes. The time from of each of the two "quiet" periods will be around 30 days. For half of the year the thermodynamic force will be from North to South. For the remaining half of the year, the thermodynamic force will be from South to North. This effect will generally agree with what is occurring in the respective Temperate Zones.

Observe that the two "quiet" periods in the Equatorial Zone are contrasted with but one "quiet" period in the respective Temperate Zones.

Observe also that the diurnal energies of the Equatorial Zone never reach the peaks of the Temperate and Polar Zones. However, The nadir’s are always greater than the diurnal energies of the respective equinoxes of the Temperate Zone.

It is from this situation in the Equatorial Zone that humans arose. The human body is adapted for natural immersion in the Equatorial Zone. Outside of this natural immersion, the human body may become very ill.

The Equatorial Zone occupies 39.8% of the surface of the Earth. The Polar Zones occupy 9.7% of the surface of the Earth. The Temperate Zones occupy the remaining 51.5% of the Surface of the Earth. The higher latitudes of the Temperate Zones are totally uninhabitable by humans without artificial contrivances.

Thus we have the general situations that generate the annual North-South winds of the Earth. A little bit of natural philosophy has been added.

The Land-Sea Breeze

The Sun warms the land. If the land is dry, the Sun warms it rapidly. If the land is moist, much of the Sun’s energy will be utilized in changing the moisture from liquid to gas; which will in turn cause the land to warm more slowly.

The Sun shines down on the sea. Much of the Sun’s energy is reflected off of the water. The energy that is absorbed by the water is quickly propelled off physically, horizontally or downward. Some of the absorbed energy is used to change the state of the water from liquid to gas. The result is the temperature of the sea is extremely resistant to change.

After the chill of the morning following sunrise, the temperature of the air over the solid Earth rises faster than the rising temperature over the sea. This results in a dynamic higher pressure over the land than over the sea. An airflow from the land to the sea is established. This effect reverses in the evening when the temperatures over the land are dropping faster than the temperatures over the sea. This effect only extends out a few miles either way from the shoreline.

For ages; Men have employed the land-sea breeze to drive their sailing vessels. A sailing vessel has a deep but slender keel that prevents the vessel from blowing cross-ways. When using the land-sea breeze, the sailors run the vessel at right angles to the wind with the sails set at an angle. Such vessels are capable of sailing faster than the wind. However, they need to stay near to the shoreline.

Scattered Clouds

The albedo of a cloud of water vapor is in excess of 65%. When the Sun shines down on a cloud from above, this radiation is reflected back into space. The albedo of the ground is normally less than 25%.

When a scattered cloud passes over a given locale, there is a sudden drop in solar radiation and a marked localized cooling under the cloud. This causes the air under the cloud to contract and decrease in volume. Meanwhile; The Sun shining on the ground outside of the cloud is collecting full solar radiation and is expanding in volume. The expanded outer air then is drawn into the contracted cooler air under the cloud. This results in a flow of air towards the shadow of the cloud.

Now comes a conflict between energy and matter. Not only must the pressure balance be equalized, so to must the energy balance be equalized.

The incoming air collides with itself under the cloud. By the path of least resistance, it forms a vortex. In the Northern Hemisphere, the vortex will have a counterclockwise rotation. In the Southern Hemisphere, the vortex will have a clockwise rotation. This will be due to a natural bias of the equatorial side moving faster than the polar side. This does not apply at the equator. The vortex of moving air around the shadow of the cloud act as a balance to the energy differential between outside of the shadow and inside of the shadow. However, this localized system has a limit and will ultimately result in a calm.

In most cases, the scattered clouds are in motion. To an observer on the ground, the wind appears to arise randomly from all directions as the is moving over the land. The motion of the scattered clouds permits the system to continuously break-down and renew itself before it has a chance to reach its limit.

So much for the origin of the Winds of the Earth. This has been a very brief and confused description of what amounts to a vast subject in itself. There are other many other effects that help to create the Winds of the Earth. There are geographical features that direct and enhance the wind. Most of these are not discussed. The sole object of this discussion is to demonstrate the direct relationship between solar energy and wind energy. Let the reader who wishes to specialize in this field seek good and reliable a-political books on the subject of meteorology.

Now let us take a look at the basic force, power, and energy equations involved with wind energy.

Some Basic Equations

The density of the air is defined as a given mass of air divided by the number of volume units that it occupies. The mass unit is commonly given as kilograms (kg) and the volume unit is commonly given as cubic meters (m^3). At sea-level the air has a mean density of around 1.27 kg/m^3.

[Mean Density of Air (ρ) = 1.27 kg/m^3]

The mass of the air is defined as the product of the density and the volume. Typically, the mass is expressed as kilograms, the volume is expressed as cubic meters, and the density is expressed as kilograms per cubic meter.

[Mass of Air = Density · Volume]

[Units = ((ρ · kg)/m^3) · (vol · m^3) = (ρ · vol) · (kg)]

However, in the case of wind energy, the effective mass of the air is measured as the volume of air enclosed in a given square unit length multiplied by the how many of those unit volumes are passing a given point in the unit time (the velocity). Typically, the unit time is expressed as the second and the velocity is expressed as meters per second. This is commonly expressed as kilograms per square meter per second.

[Effective Mass of Moving Air = density · velocity]

[M = ρ · (kg/m^3) · v · (m/s) = (ρ · v) · ((kg/m^2)/s)]

Kinetic Energy is expressed as one-half of the product of mass and the square of the velocity,

[E_k = (½) · (M · kg) · (v^2 · (m^2/s^2))]

However, for wind energy there is a twist. This problem also brings up an issue concerning the tendency to over-simplify an equation. If the kinetic energy equation is fully simplified, then the resulting units will be [kg/s^3] which is not reasonable. The over-simplification occurred when the physical situation was ignored. The division by the square of the unit length represented the applied unit cross-section of the moving air. Therefore, it must be reintroduced.

$$[E_k = (½) \cdot (M \cdot kg) \cdot (v^2 \cdot (m^2/s^2))]$$

$$[E_k = (½) \cdot ((\rho \cdot v) \cdot (kg/m2)/s))) \cdot (v^2 \cdot (m^2/s^2))]$$

$$[E_k = (½) \cdot (\rho \cdot v^3) \cdot (kg/s^3) = (½) \cdot (\rho \cdot v^3) \cdot (kg \cdot (m^2/s^3)/m^2)]$$

Because this variant of the kinetic energy equation utilizes the velocity of the air as the independent variable, it must be considered to be reflective of the power of the wind for an area of one square meter at right angles to the direction of the wind. This is because the velocity assumes a variable length and a constant unit time of one second. In short, it represents the events occurring in the unit time of one second. Observe that the power of the wind as demonstrated varies as the cube of the velocity of the wind.

The work done by the wind then must be represented by the product of the power and time of application.

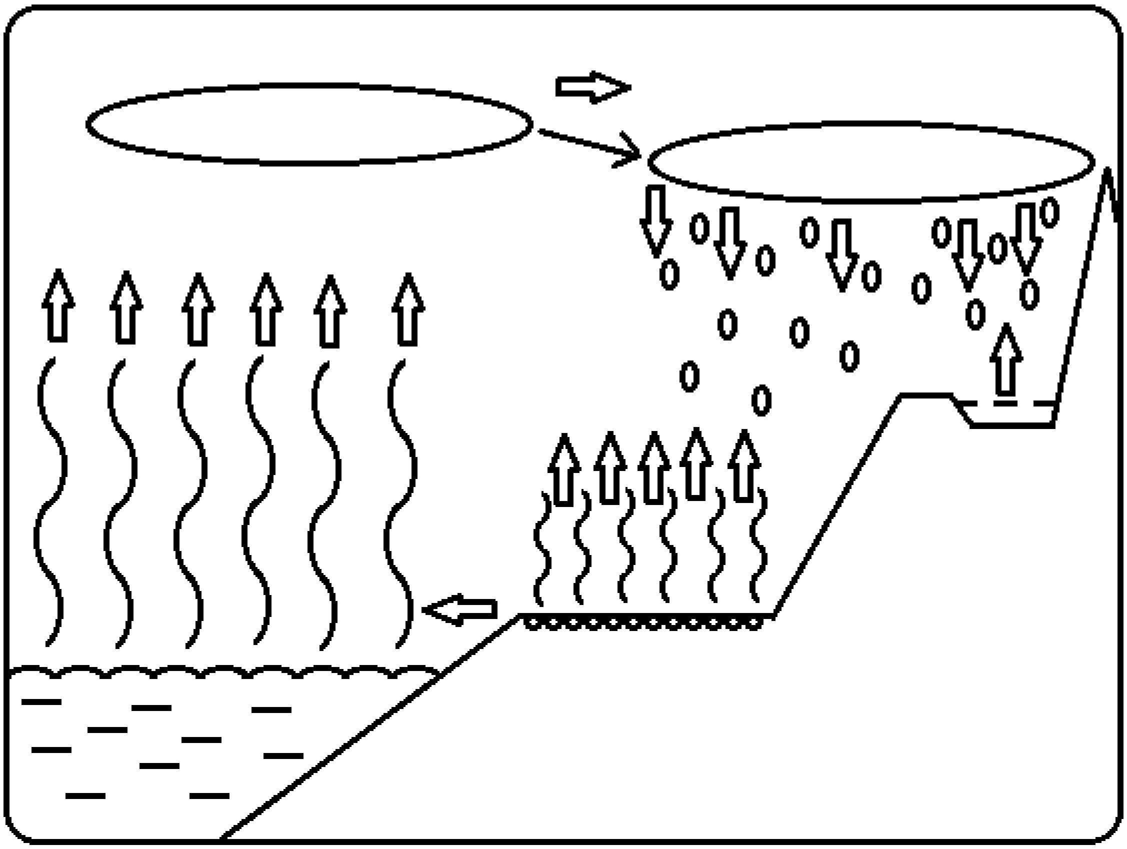
$$[Work = ((½) \cdot \rho \cdot v^3 \cdot t) \cdot (((kg \cdot m^2)/s^2)/m^2)]$$

Let us now examine what happens when the power is divided by the velocity. Observe that the unit analysis of the result indicates a force of the wind. Also observe that this force of the wind varies as the square of the velocity.

$$[F = ((½) \cdot \rho \cdot v3)/v = ((½) \cdot \rho \cdot v3) \cdot (((kg \cdot m)/s^2)/m^2)]$$

Chapter XII

The Waters of The Earth



A Brief Discussion of the cycle of the waters of the Earth.

The rays of the Sun warm the land of the Earth. A portion of that energy is used to warm (energize) the seas of the Earth. The rays of the Sun warm the seas of the Earth directly as well. Vapor rises into the air and transfers its heat to the surrounding air. In the process, clouds are formed. The clouds travel outward and precipitate their moisture onto the land and the sea. The moisture received by the land is both drained into the sea or rises into the air as vapor. This is the cycle of the waters of the Earth.

The Real Hollow Earth Theory

The writer of this book is a follower of the theory that the Earth is a hollow sphere, as well as the other planets. Let be clear on this. There are two basic Hollow Earth theories. One theory is clear and concise while the other theory is a vague tool for promulgating social doctrines. This brief discussion will deal with the former. This does have a relevance to the Waters of the Earth as well as a few other important topics.

Ages ago a mass of no significant size passed through a cloud of dust and gases left behind by an exploding star. Within a certain radius of the insignificant mass, matter was drawn in, that is, the matter was “falling”, towards the insignificant mass. As the matter coalesced in the vicinity of the insignificant mass, the incoming matter became a significant mass and the insignificant mass was either absorbed into the now significant mass or simply departed to continue on its long journey.

The incoming matter approached the center near escape velocity from all directions. The incoming matter conflicted with other incoming matter resulting in a decrease in velocity and the generation of a lot of heat.

There is always a predilection in any conflict for the establishment of a reasonably coherent order. So it was in the beginning. However, matter in motion cannot “fall” through other matter “falling” the other way. Thus, the center was left empty, while a hot spinning disk formed around the empty center. The point of zero gravity of for this spinning “ring” was a point about a quarter of the way up with respect to the thickness of the ring. The center of attraction was a ring a little more than a half of the way up with respect to the thickness of the ring.

Matter continued to fall inwards onto the center of attraction of the spinning ring forming a hollow sphere. As the sphere built up, the point of zero gravity moved inwards. When the sphere was completed, the point of zero gravity was located at the inner surface of the hollow sphere. However, the new planet still had a high rotation rate and a considerable outward centrifugal force.

The writer of this discussion has mathematically obtained the following three general conclusions about hollow worlds: **[1]** All points within the shell of a hollow sphere have a zero gravity. **[2]** The rate of inward attraction due to gravity within the shell of the hollow world generally declines linearly from the outer surface to zero at the inner surface. **[3]** The rate of acceleration due to gravity outside of the body of the hollow world always declines with respect to the square of the distance from the center of the hollow sphere.

Due to the varying rotational velocities within the shell of the hollow world, stress forces are produced. Where these stress forces become too great, there are violent separations which produce a great deal of heat. These separations form “gravelly” discontinuities.

The first and most obvious discontinuity is the polar axis. The fanatics would have a gaping polar opening leading to a “paradise” of their own imagined desires. This is nonsense. However, The opposing directions about the polar axis would lead to a sort of gravelly fissure or tunnel, open at the bottom, but naturally diked over at the top by debris falling in from the surface. The debris dike would be many miles deep.

Within the body of the shell of the hollow world, a complex of other discontinuities would form. Sometimes a slippage will occur, resulting in heat, earthquakes, and volcanic action.

A hollow world is maintained by horizontal attraction within the shell itself. At the outer surface, the force vector will be at an angle downwards. At the inner surface, the force vector will horizontal. At the inner surface there is no inclination to fall or compress further inwards. How strong is this system you may ask? The writer of this discussion has done an experiment with an incandescent light bulb. A burned out light bulb was employed. There was a pressure of 14.7 lbs psi acting on the thin glass surface of the bulb, maybe around 100 lbs total. The burned out light bulb was hit with a 16 oz hammer. The hammer just bounced back without breaking the thin glass shell. Then the little tab at the base of the bulb was broken, letting in air. The experiment was repeated, and the thin glass was crushed like an eggshell. The shell of a hollow world is far thicker than a light-bulb, and the loading pressure due to gravity per thickness is less than 1:100,000 with respect to the light-bulb.

The shell of a hollow world is not wholly solid matter. This is a lesson that was brought home to the writer of this discussion while engaged in digging post-holes and setting fence-posts. Whenever you dig a post-hole and then set a fence-post in the hole; When you fill and pack the hole you always observe that the whole of the replaced earth always just fills the post-hole with the fence-post inserted. This means that beneath the surface, the solid matter only occupied between 50% and 75% of the volume of earth that was removed from the hole.

Within the shell of a hollow world, there is a vast series of tunnels and caves. The “land-area” probably far exceeds the combined inner and outer land-areas. This is at the macro-level. The same thing occurs at the micro-level. Packed random gravels, random sands, and random crystals do not perfectly mesh with one-another. This leaves pores to be filled with gases or fluids, including water. The surface of this porous network tends to get “diked-up” by surface debris falling down into the upper pores, causing it to look solid. Think in terms of a jar filled with marbles; and then some sand is poured over the top, filling the upper porous surface, but leaving the lower portions undisturbed.

For our discussion on the Waters of the Earth with respect to solar energy, we are mainly concerned with waters that lie over the diked-up ocean and lake beds, and with the waters that have penetrated and filled the surface pores of the land.

The warming of the Waters of the Earth

The Sun shines down upon the land. The land is heated by the energetic rays of the Sun. The energetic rays of the Sun pass through the air. The air is heated by the energetic rays of the Sun. The Sun shines down on the sea. The waters of the sea are warmed by the energetic rays of the Sun.

The surface waters of the land are warmed by the heated land. The surface waters of the land are warmed by the heated air. The waters of the sea are warmed by the heated land. The waters of the sea are warmed by the heated air.

A Mist rose from the Sea and from the Land

The heating of the sea is mostly limited to the surface. At the bottom of the sea, the temperatures are near freezing. However, we are not concerned with the bottom of the sea. We are only concerned with the surface of the sea.

Just beneath the surface of the land, the ground is porous. These pores may constitute anywhere from 25% to 50% of the total volume of the ground. Normally these naturally occurring pores are filled with water. This is the water that feeds the living plants. There are deeper waters under the land, but those deeper waters are not part of this discussion. We are only concerned with the surface waters of the land.

The surface waters of the sea maintain a more or less steady temperature. The temperature of the surface of the sea produces a water vapor pressure in the local air that rises from the sea. During the heat of the day, it may only be observed as a haze in the air. In the cool of the night and in the chill of the morning hours just after sunrise, it may be observed as a mist rising from the sea.

The temperature of the surface waters of the land are more volatile than the surface waters of the sea. However, the surface of the land contains a great amount of stabilizing thermo-mass. During the early morning hours a mist will appear to rise out of the surface of the land as it is heated by the return of the Sun. This mist is the result of the waters in the just heated surface pores rising as a vapor and then condensing due to the chill of the morning air. In the heat of the day it will appear as a haze in the air.

Clouds form over the Sea and over the Land

The process of heating the waters of the Earth has injected work energy from the Sun. This results in water vapor over the sea and the land. Since water vapor has but 60% of the density of air, the water vapor rises. As the warm water vapor rises, by contact or thermo-radiant energy, it passes its excess temperature to the surrounding air. In the process it condenses into ultra-fine water droplets.

When this condensation happens close to the sea or the ground, it is known as fog. When this condensation occurs at a higher elevation, it is known as clouds.

The Clouds move Inland: Precipitation

The clouds drift with the wind. If the clouds are located in, or encounter, a mass of colder air, there will be sudden condensation of the clouds resulting in precipitation back down to the sea or to the ground. When the clouds move inland and encounter an uplift in the lay of the land; due to the decreasing pressure there will be a cooling and condensation resulting in precipitation to the ground.

Fog occurs when the water vapor condenses near the surface. Fog acts as a contact precipitate on plants, animals, and the Earth. Likewise for the dew. The main difference between the dew and the fog is that the dew is more marginal and the fog stage is less likely to be observed, normally occurring in the chill of the morning before daybreak when most people are still asleep.

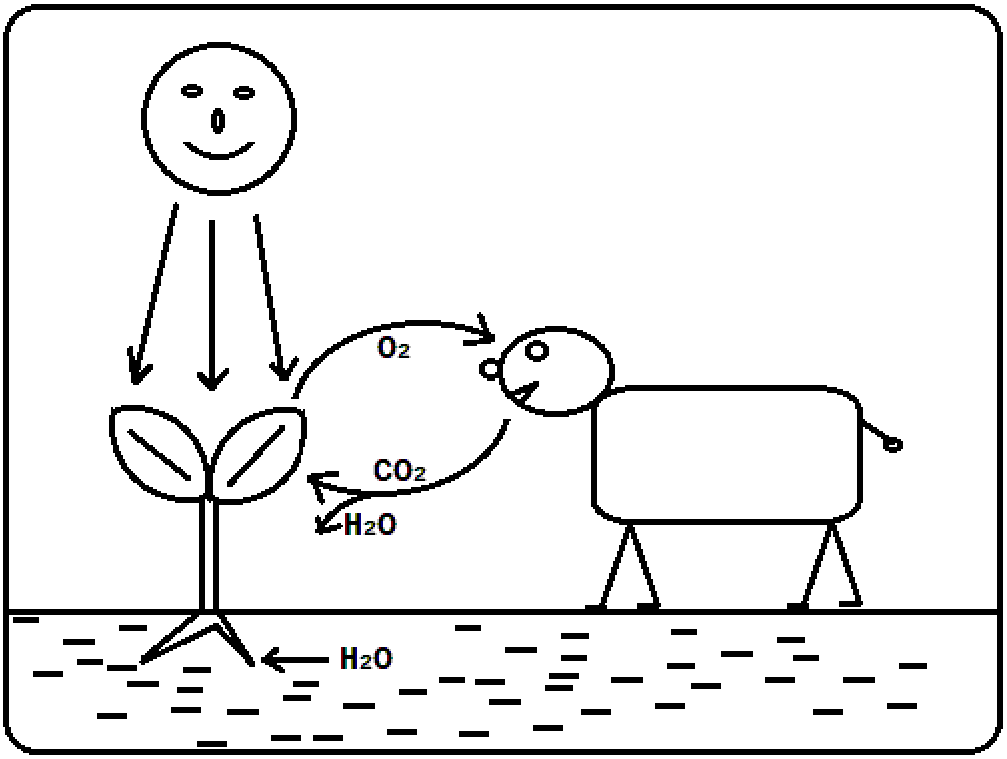
The formation of clouds during the day results in an increased local albedo with a corresponding decrease in received radiation. The presence of clouds during the night results in a decrease in local thermo-radiant energy. The former results in a cooler temperatures during the day and the latter results in milder temperatures during the night.

Hydropower

When the precipitation of water from the clouds falls onto higher ground, a hydraulic energy potential is established between the higher ground and the lower ground. This hydraulic energy potential is fully realized when the water precipitated onto the higher ground is returned to the sea. In the process; marshes, creeks, rivers, lakes, and waterfalls are created.

Chapter XIII

Plants and Animals of the Earth



A Very Brief Look at the Sun's Effect on Biochemistry

Photosynthesis

Photosynthesis is the process by which a green plant containing chlorophyll initiates the process that produces all carbon-based organic life on the Earth.

The green plants draw up water through their roots up into the green leaves. Meanwhile, carbon-dioxide is being taken in from the air into the green leaves. Through the processes in the chlorophyll, the water is broken down into hydrogen and oxygen. The oxygen is released to the atmosphere while the hydrogen is held back to combine with the carbon-dioxide.

This is the basic process by which all of the oxygen in the atmosphere of the Earth is obtained. Here are two common variants of the theme.

$$CO_2 + 2H_2O + ☼ \rightarrow CH_2O + O_2\uparrow$$

$$6CO_2 + 12H_2O + ☼ \rightarrow C_6H_{12}O_6 + 6O_2\uparrow + 6H_20\uparrow$$

Under certain conditions Sulfur from Hydrogen-Sulfide may substitute for water in the light reaction. This happens in swamps and around thermal vents on the ocean floor out of reach of the light of the Sun. In the latter case, the radiant energy is supplied by the Earth itself. In the presence of oxygen, the water reaction will always preclude the Hydrogen-Sulfide reaction. Here is a reaction that has been demonstrated by “sulfur bacteria” that is identical to the first of the preceding reactions.

$$CO_2 + 2H_2S + ☼ \rightarrow CH_2O + S_2\uparrow$$

The second case is more elaborate and will not substitute for sulfur. That is not to say that a similar reaction is not possible.

This has been the case for the Earth that we know under the Sun that we know. Other light reactions may also occur beneath the surface of our world or in other worlds and stars. Within a world, out of the light of the parent Sun, radiation is still present. It may have a different wavelength, but that does not mean that it will either be feebler or stronger. One such possible substitution is replacing carbon with silicon as follows:

$$SiO_2 + 2H_2O + \text{☼} \rightarrow SiH_2O + O_2\uparrow$$

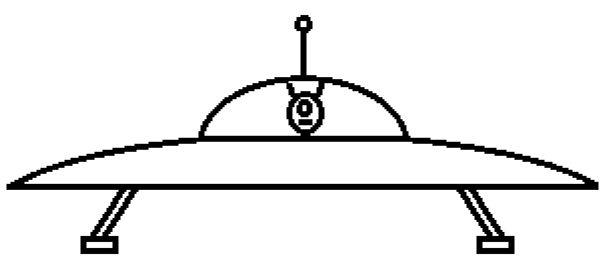
$$6SiO_2 + 12H_2O + \text{☼} \rightarrow Si_6H_{12}O_6 + 6O_2\uparrow + 6H_2O\uparrow$$

OR

$$SiO_2 + 2H_2S + \text{☼} \rightarrow SiH_2O + S_2\uparrow$$

Due to the requisite high temperatures, the latter would be a good choice for writing an imaginative alternative science fiction story about life in a star or in a close orbit about a star.

There are many other possibilities as well. They will be found in the periodic table of the elements.



Vegetation of the Earth (Flora)

This chapter has not been written, or intended, as a primer on biochemistry. Biochemistry is a vast subject on which many excellent books have been written; Books written by men and women whose entire lives have revolved about the subject.

With the aforementioned disclaimer out of the way, let us continue on with this brief discussion on the flora of our Earth. Metaphysical philosophy will be involved in the discussion.

When light is received by a photosynthetic bacteria, there occurs a simple reaction that either breaks down water (h_20) or Hydrogen-Sulfide (H_2S) respectively in order to “fix” Carbon Dioxide (CO_2). The final product is O_2 or S_2 respectively and CH_2O in all cases. This will be followed by any number of subsequent internal (dark) reactions necessary for the life of the respective bacteria. This is shown in the preceding section.

In the case of plant life, the light is received by specialized cells called “chloroplasts.” Here the (light) reactions are more complicated. A simple case is also shown in the preceding section. This too, is followed by a series of subsequent internal (dark) reactions.

From these (light) and (dark) reactions is produced the whole of the flora of the Earth. These are very complicated and very involved reactions. The complexity of these reactions is akin to precision backing a triple trailer; or calculating a three-body astronomical equation.

There are some that say the process was created by some God in an eye-blink. There are others that say that it was initiated billions of years ago by some accident of nature, and natural evolution completed the process. The reality undoubtedly lies between the two extremes; originating on this Earth of ours, or some other planet.

Fauna and Fire

The vegetable kingdom (flora) is a vast primary empire. All of the trees and the grasses are continuously being reproduced upon the face of the Earth. If this reproduction were to continue indefinitely without any checks, all of the available carbon-dioxide would soon be consumed and all new plant growth would become an impossibility.

However, there is a check. Animals (fauna) and fire. The materials within the plants represent a chemical energy imbalance with respect to the oxygen in the air, the same oxygen that was driven out by the initial reaction to the Sun.

Animals consume the plants and produce water and carbon dioxide. The cycle is completed. Here is a simple example of this completion taken from the preceding example of photosynthesis.

$$6CO_2 + 12H_2O + \text{☼} \rightarrow C_6H_{12}O_6 + 6O_2\uparrow + 6H_2O\uparrow$$

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2\uparrow + 6H_2O\uparrow + \text{Energy}$$

Thus the cycle is complete. However, if the animals continue to reproduce while feeding on the plants, ultimately they will meet with a check themselves when they are consuming more plants than the plants can reproduce. In the end, both will be diminished.

The natural solution is predators that feed on other animals. They are necessary to keep the animals healthy. Some predators feed on other predators.

In the end all of the older plants and animals must die as well as the weak degenerate plants and animals. The smallest of the predatory animals complete their return to the Earth, the dark, non-photosynthetic bacteria.

Sometimes this natural process is aided by the incursion of fire. Fire helps to cleanse the Earth of life, both plant and animal, that is overstepping its bounds.

It is estimated that the energy storage capacity of plant life is in the neighborhood of around 1%. Plants and animals use this 1% in a closed cycle. Humans are only a tiny part of the cycle and are assigned but a small share of garnered energy of the Sun through the plant and animal life.

Chapter XIV

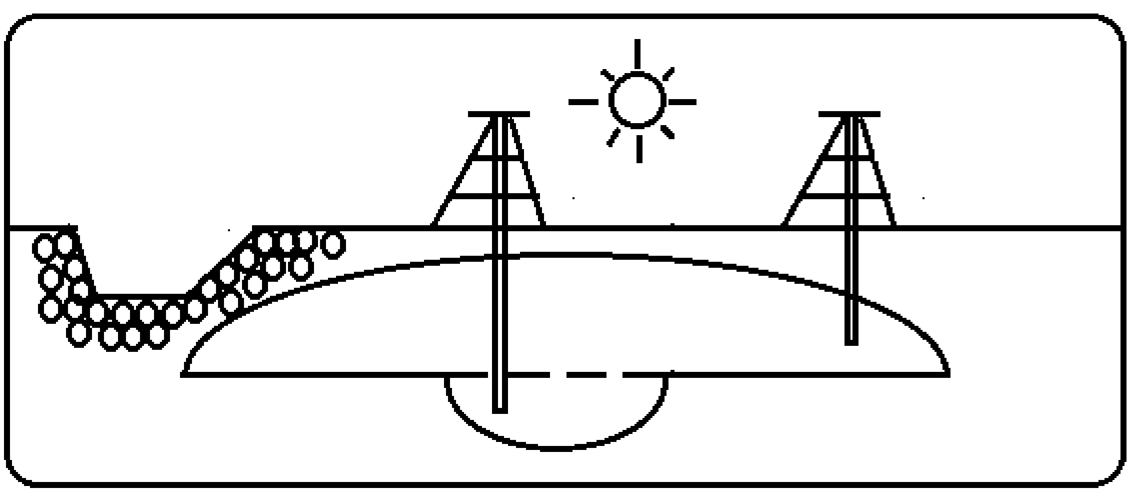
The Unholy Trinity

Coal, Petroleum, Natural Gas

Three Rings for the Elven-kings under the sky,
Seven for the Dwarf-lords in their halls of stone,
Nine for Mortal Men doomed to die,
One for the Dark Lord on his dark throne
In the Land of Mordor where the Shadows lie.
One Ring to rule them all, One Ring to find them,
One Ring to bring them all and in the darkness bind them
In the Land of Mordor where the Shadows lie.

J. R. R. Tolkien

Taken from the 'Lord of the Rings' with Respect



This subject of the “Unholy Trinity, Coal, Petroleum, Natural Gas” has not been written as a lesson nor as a history of these extremely complex subjects. Many excellent books have been written specifically about every aspect of this subject except one; the end-user solar energy connection. Here then is the final chapter in the story.

The origins of coal, petroleum, and natural gas are shrouded in the mists of time. There are many excellent theories about the origins as well as some unlikely theories. The jury is still out on which is which. It may turn out that all of the theories will ultimately prove to be correct, each in its own place.

Coal, oil, petroleum, and natural gas all come from under the Earth, out of sight of the Sun, the proverbial underworld of the Devil, hence the reference to the “Unholy Trinity.” The link between them is that their utilization requires oxygen from the atmosphere. That oxygen is supplied by green leaves irradiated by the light of the Sun. Thus, these three classes of “petrochemicals” are taken as “gods” by the inhabitants of the Earth who never take notice that the required oxygen is produced by the Sun, the true ruler of the Earth. This is a classic application of evil; to offer something at a price that is freely available to all; and then to claim the credit!!!

The infrastructure required to use the petrochemicals is vast and extremely complicated. The petrochemicals have been mostly sealed underground for ages. Due to the principles of adaptive evolution, the petrochemicals are toxic to all living things on the surface of the Earth. All of the petrochemical components brought to the surface must fully utilized and converted to non-toxic carbon-dioxide and water. This process of full utilization creates a vast entrapping demonic infrastructure that has the Devil laughing and dancing on the streets of petrochemical asphalt!!!

Natural Gas

A common theory about natural gas is that it comes from the methane decay of coal and petroleum. This theory postulates that the coal and petroleum represent the mortal remains of long dead plants and animals. Another alternate theory suggests that the natural gas was created deep under the Earth from the compressing of the primordial methane at the day of creation. Given the extended time frame; both theories most likely apply.

Generally speaking, natural gas consists of six hydrocarbon components. They are structurally referred to as "polymers". A "mer" is given by the empirical equation of [CH_2]. Here are the six applicable polymers of natural gas with their attendant structures. Also shown is the applicable methane decay and the combustion parameters:

[C0] Hydrogen Gas H_2 --> No structure required.

$2H_2(4g) + O_2(32g) \rightarrow 2H_20(36g)$

$O_2/2H_2 = (32g/4g) = 8$

[C1] Methane Gas CH_4 --> $2H + 1 \cdot (CH_2)$.

$2CH_4(32g) + 4O_2(128g) \rightarrow 2CO_2(88g) + 4H_20(72g)$

$4O_2/2CH_4 = (128/32g) = 4$

[C2] Ethane Gas C_2H_6 --> $2H + 2 \cdot (CH_2)$.

$2C_2H_6(60g) + 7O_2(224g) \rightarrow 4CO_2(176g) + 6H_20(108g)$

$7O_2/2C_2H_6 = (224g/60g) = 3.7333…$

Ethane Decay: $C_2H_6 - Ch_4 \rightarrow C + H_2$.

[C3] Propane C_3H_8 --> $2H + 3 \cdot (CH_2)$.

$2C_3H_8(88g) + 10O_2(320g) \rightarrow 6CO_2(264g) + 8H_20(144g)$

$10O_2/2C_3H_8 = (320g/88g) = 3.6363…$

{At this point the methane release from the petrochemicals are going to result in a complex mixture of polymers and other forms.}

[C4] Butane C_4H_{10} --> 2H + 4 · (CH_2).

$2C_4H_{10}(116g) + 13O_2(416g) \rightarrow 8CO_2(352g) + 10H_20(180g)$

$13O_2/2C_4H_{10} = (416g/116g) \approx 3.5862$

These are the basic components (polymers) for natural gas. Observe that in all cases over three times as much green chlorophyll plant derived oxygen is required for the utilization. The energy source for this oxygen is the Sun shining down on the already existing carbon based plants.

Butane has a boiling point of 0ºC. This is the freezing point of water. Beneath the surface of the Earth the polymers butane and lower will be in a compressed gaseous state.

In the natural world where the surface is open, these “natural” gasses will slowly seep to the surface where they will be acted on by natural bacteria producing primordial carbon-dioxide. This primordial carbon-dioxide is what maintains life on the surface of the Earth. In the event of a catastrophic solar storm, this continuous seepage of natural gases will rebuild the atmosphere. It has happened many times.

However, “natural” gas is but a servant for the Earth. The Sun is still the master over the Earth. A servant is always inferior to the master; and when the servant tries to usurp the master, degradation and destruction will always follow. This is why “natural” gas has been named as a member of the “unholy trinity.”

Observe that “natural” gas being a gas, may travel far and seep into far removed cavities, gravels, and shales all over the world.

Petroleum

Beneath or far removed from the natural gas deposits are the higher polymers commonly known as petroleum. They generally range from [C5] to [C14]. The following is a list of the formulae and reactions of the respective polymers commonly found in petroleum. Sometimes these seep to the surface as well, pardon the unintentional pun, and are acted on by bacteria as “well.”

[C5] Pentane C_5H_{12} --> 2H + 5 · (CH_2).

$2C_5H_{12}(144g) + 16O_2(512g) \rightarrow 10CO_2(440g) + 12H_20(216g)$

$16O_2/2C_5H_{12} = (512g/144g) = 3.5555…$

[C6] Hexane C_6H_{14} --> 2H + 6 · (CH_2).

$2C_6H_{14}(172g) + 19O_2(608g) \rightarrow 12CO_2(528g) + 14H_20(252g)$

$19O_2/2C_6H_{14} = (608g/172g) \approx 3.5349$

[C7] Heptane C_7H_{16} --> 2H + 7 · (CH_2).

$2C_7H_{16}(200g) + 22O_2(704g) \rightarrow 14CO_2(616g) + 16H_20(288g)$

$22O_2/2C_7H_{16} = (704g/200g) = 3.52$

[C8] Octane C_8H_{18} --> 2H + 8 · (CH_2).

$2C_8H_{18}(228g) + 25O_2(800g) \rightarrow 16CO_2(704g) + 18H_20(324g)$

$25O_2/2C_8H_{18} = (800g/228g) \approx 3.5088$

[C9] Nonane C_9H_{20} --> 2H + 9 · (CH_2).

$2C_9H_{20}(256g) + 28O_2(896g) \rightarrow 18CO_2(792g) + 20H_20(360g)$

$28O_2/2C_9H_{20} = (896g/256g) = 3.5$

[C10] Decane $C_{10}H_{22}$ --> 2H + 10 · (CH_2).

$2C_{10}H_{22}(284g) + 31O_2(992g) \rightarrow 20CO_2(880g) + 22H_20(396g)$

$31O_2/2C_{10}H_{22} = (992g/284g) \approx 3.4930$

[C11] Undecane $C_{11}H_{24}$ --> 2H + 11 · (CH_2).

$2C_{11}H_{24}(312g) + 34O_2(1088g) \rightarrow 22CO_2(968g) + 24H_20(432g)$

$34O_2/2C_{11}H_{24} = (1088g/312g) \approx 3.4872$

[C12] Dodecane $C_{12}H_{26}$ --> 2H + 12 · (CH_2).

$2C_{12}H_{26}(340g) + 37O_2(1184g) \rightarrow 24CO_2(1056g) + 26H_20(468g)$

$37O_2/2C_{12}H_{26} = (1184g/340g) \approx 3.4824$

[C13] Tridecane $C_{13}H_{28}$ --> 2H + 13 · (CH_2).

$2C_{13}H_{28}(368g) + 40O_2(1280g) \rightarrow 26CO_2(1144g) + 28H_20(504g)$

$40O_2/2C_{13}H_{28} = (1280g/368g) \approx 3.4783$

[C14] Tetradecane $C_{14}H_{30}$ --> 2H + 14 · (CH_2).

$2C_{14}H_{30}(396g) + 43O_2(1376g) \rightarrow 28CO_2(1232g) + 30H_20(540g)$

$43O_2/2C_{14}H_{30} = (1280g/368g) = 3.4747\ldots$

[Cn] [Latin] + [ane] $C_nH_{(n + 2)}$ --> 2H + n · (CH_2)

As can be be seen, the components of petroleum require on the average around 3.5 times as much solar energy derived oxygen as petroleum in order to utilize the petroleum. The petroleum sometimes seeps to the surface where it is acted on by bacteria or is consumed by flames as an “eternal flame.” In either case, the slow seepage and the resultant production of carbon-dioxide serves to balance against the slow loss of atmosphere to space.

Coal

Coal is a confused mess!!! The origins of coal are a totally confused mess of natural gas, petroleum, and ancient plants and animals buried by innumerable long forgotten past cataclysmic geological events. For these arguments we will examine the latter situation.

In the natural world all plant and animal life dies and decays producing water, carbon-dioxide, and methane. The methane is acted on by bacteria or consumed by fire producing water and carbon-dioxide. The water and the carbon-dioxide are then acted on by the Sun via chlorophyll producing complex carbon-based life forms and atmospheric oxygen.

During a cataclysmic event, the plants and the animals get deeply buried in the debris. This disrupts the natural cycle. In order to recover, new plants and animals are produced from the remaining atmospheric carbon-dioxide. This draws down and places an additional constraint on how much life the Earth can support. Meanwhile, below the surface of the Earth, the buried plant and animal life is undergoing different forms of decay. These alternative forms of decay produce the near-surface coal deposits.

Sometimes the coal deposits will be naturally exposed and ignited, such as the “Franklin Cliffs” in the Arctic. This serves to return lost carbon-dioxide to the atmosphere.

The issue with using any of these petrochemicals is that they are intended by nature to be rationed. If they are exploited, the following cataclysmic even could mean the total extinction of all life on the Surface of the Earth rather than a simple, howbeit extremely unpleasant, inconvenience.

With reference to the Ring Poem; The exploitation of the petrochemicals constitutes an insidious trap for the unwary!!!

Conclusion

Living Under the Sun

The Environmental Survey

From the preceding chapters of this book; It is clear that the only source of life, sustenance, and wealth in this World is the light of the Sun acting on the Earth. This is our only resource. However, this resource comes in many forms. In order to properly live under the Sun, all of the forms must be accounted for.

Enter the environmental survey. The environmental survey is a specific site survey. It includes an “obstruction” altitude” survey, an historical weather survey, a soil survey, a water survey, a flora (plant) survey, and a fauna (animal) survey. These six items are then bound up in an environmental site report. This is an objective report that has nothing to do with politics or religion. It is a working reference.

The “Obstruction” Altitude Survey

This survey is an easy one, howbeit rather tedious. A transit is set up on the site with an azimuth and altitude reading. A line is shot from site to the top of an obstacle on the horizon. The azimuth angle and the altitude angle are then recorded from the transit. For really good equipment this survey should be done in 1º increments with respect to the azimuth angle. This will results in 360 pairs of azimuth-altitude data. Otherwise, 5º increments of azimuth could be used resulting in 72 pairs of azimuth-altitude data. It is inadvisable to use azimuth increments greater than 5º.

The results from the “obstruction” altitude survey will be used in determining the “obstruction” angle to be used in the solar power and accumulated solar energy calculations. It will also provide information on the times that the site may experience direct sunlight for other purposes. Of course, there may be times when shade is required. This latter will also be provided for.

The Historical Weather Survey

This survey is a much more difficult one. Unlike the preceding survey, the historical weather survey will not yield fixed data for all times. Instead, it will provide seasonal trends that will vary by probability.

The historical weather survey requires the use of weather instruments. It also requires the use of a logbook to record the readings. The readings should be taken at least every hour of every day of the year. At least three years are needed to get a general seasonal trend. However, not every site needs a weather station. A proper historical weather survey may be done as a local community project where all share in the results.

Here are some recommended measurements:

[01] Accumulated Precipitation.

[02] Barometric Pressure.

[03] Air Temperature at 20′ above ground.

[04] Dew-point at 20′ above ground.

[05] Blackbody temperature at 20′ above ground.

[06] Wind Velocity at 20′ above ground.

[07] Wind direction at 20′ above ground.

[08] Air Temperature at 6′ above ground.

[09] Dew-point at 6′ above ground.

[10] Blackbody temperature at 6′ above ground.

[11] Wind Velocity at 6′ above ground.

[12] Wind direction at 6′ above ground.

[13] Temperature on surface of ground exposed to Sun.

[14] Temperature on surface of ground not exposed (shaded) to Sun.

[15] Temperature 12″ below surface of ground.

[16] Dew-point 12″ below surface of ground.

[17] Temperature 72″ below surface of ground.

[18] Dew-point 72″ below surface of ground.

[19] Solar radiation for level surface.

[20] Magnetic Declination Angle.

[21] Magnetic Dip angle.

[22] Electrical Charge of atmosphere.

[23] North-South Horizontal component of geologic activity.

[24] East-West Horizontal component of geologic activity.

[25] Up-Down Vertical component of geologic activity.

[01] through [19] are more or less the standard measurements. [20] through [25] are more or less auxiliary measurements. All of these will provide useful information on what may be expected of the site. The historical weather survey is a tool for planning.

In addition; A journal should be kept to record the date and time of irregular events as well as such things as snowfall, moisture percentage of snowfall, and singular natural meteorological events.

The Soil Survey

The soil survey is both specific to the site and general for the local community.

It is often needful to know the details of the local soil. This includes the geology of the site, the minerals contained in the soil, and the acidity of the soil.

Sometimes soil will contain toxins. Sometimes the toxin is naturally occurring. Sometimes the toxins are the result of human activities.

The soil should also be examined for microbes, bacteria, and virus' that may effect plant life.

The Water Survey

In any area there will be bodies of water. The waters will contain minerals, chemicals, and microorganisms; both natural and introduced by humans or the livestock of humans.

The local waters should be charted with seasonal variations. The seasonal temperatures for varying depths should be recorded. This includes lakes, reservoirs, streams, creeks, springs, and wells.

A mineral and micro-biological examination of the waters should be conducted.

Flora (Plants)

It is essential to identify the local plant life grow in the area of the site, both land and aquatic, both native and introduced.

Fauna (Animals)

It is equally essential to identify the local animal life in the area, both land and aquatic, both native and introduced.

Epilogue

This book was completed on Thu 08 Dec 2022. At this time, there have been great signs appearing in the Heaven above. The vast majority of humanity have never looked up to observe them. At this time, there have been heard rumors in the Earth below. Few humans have bothered to place their ears to the ground to listen. At this time, there have been great wars and rumors of greater wars to come. The bulk of humanity has only been unquestionably concerned with preserving their way of life.

These are legendary times. Great changes are in store for all the Earth. There is a great war unraveling, a war that began many ages ago. The Generals are positioning their pieces on the board in time so that all converge at the same time for the great battle. Those who dwell below the Earth, those who dwell on the Earth, and those who dwell above the Earth will meet in the great battle of this age.

The meek shall “inherit” the Earth. The meek will live under the light of the Sun and reject the shadow-lands of the arrogant. This book is dedicated to the meek of the Earth. Many of you may already be born to rebuild the Earth in a better way after the the destruction of the arrogant!